OPSC
OAS

Previous Year Paper

Mains 2019

(Mathematics) Paper-II

CSM – 53/19

Mathematics

Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Attempt any **three** of the following :

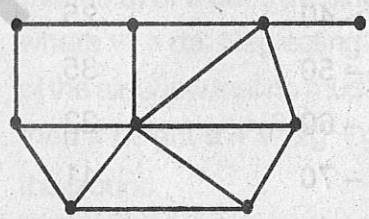
   (a) (i) The following table gives the marks secured by 100 students in mathematics : 10

Range of Marks	No. of Students
30 – 40	25
40 – 50	35
50 – 60	22
60 – 70	11
70 – 80	7

Using Newton's forward difference interpolation, find the number of students who secured marks in the range from 36 to 45.

(ii) The polynomials $\phi_0(x) = 1$, $\phi_1(x) = x - \frac{1}{2}$, $\phi_2(x) = x^2 - x + \frac{1}{6}$ are orthogonal in [0, 1]. Using these polynomials find the least square polynomial approximations of degree 0, 1, 2 for the function $f(x) = x^2 - 2x + 3$ on [0, 1]. 10

(b) Determine whether the graph as shown below is a simple graph, multigraph or pseudograph. Find the degree of each vertex. Also find a simple path. 20



(c) (i) Solve : 10

$$2x^2\frac{dy}{dx} = (x-1)(y^2-x^2)+2xy$$

(ii) Solve : 10

$(x^2D^2 + 4x + D + 2)y = \sin x$

(d) Solve : $y'' - 2x^2y' + 4xy = x^2 + 2x + y$ in powers of x. 20

2. (a) (i) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by Regula-Falsi method correct upto three places of decimal. 15

(ii) Give the geometrical interpretation of Newton-Raphson method and find the rate of convergence of this. 15

(b) (i) Using Euler's modified method solve numerically the equation $y' = x + |\sqrt{y}|$ with $y(0) = 1$ for $0 \le x \le 0.6$ in step of 0.2. 15

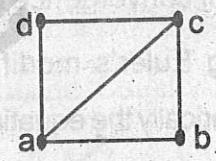
(ii) Solve the equation $y' = x + y$ with $y_0 = 1$ by Runge-Kutta rule from $x = 0$ to $x = 0.4$ with $h = 0.1$. 15

3. (a) (i) Define a bipartite graph. A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color. Prove this. 20

(ii) What is the minimum number of vertices in a 3-regular bipartite planer graph ? 10

(b) (i) A Tree has 3 vertices of degree 3 each. What is the number of leaves in this tree ? 20

(ii) Draw the spanning trees of the following graph : 10



4. (a) (i) Solve $Dx + Dy = t$, $D^2x - y = e^{-t}$ using Laplace transform method which satisfies the conditons $x(0) = 3$, $x'(0) = -2$, $y(0) = 0$. 15

(ii) Solve $y^2z(x\ \cos x - \sin x)dx + x^2z(y\ \cos y - \sin y)dy + xy\ (y\ \sin x + x\ \sin y + xy\ \cos z)dz = 0.$ 15

(b) (i) Find a complete integral of $p^2x^2 = z(z - qy)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. 10

(ii) Reduce $x^2r - y^2t + px - qy = x^2$ to canonical form. 20

SECTION – B

5. Answer any **three** of the following :

(a) Write a flow chart and a FORTRAN program to find the largest of three given numbers A, B, C. 20

(b) A ladder of weight W rests on an angled to the horizontal with its end resting on a smooth floor and against a smooth vertical wall. The lower end is joined by a rope to the junction of the wall and the floor. Find the terms of W and α, the tension the rope and the resistance of the wall and the ground. 20

(c) Find the dual of the following LPP : 20

$$\text{Min } z = 2x_1 - 4x_2 + 3x_3$$

s.t. $2x_1 - 2x_2 + 3x_3 \leq 8$

$4x_1 + 4x_2 + 10x_3 \leq 20$

$3x_1 - 5x_2 \leq 10$

x_1, x_2 are unrestricted and $x_3 \geq 0$.

(d) Derive Euler's Equation of motion in Cartesian form. 20

6. (a) Using the dual, solve the following LPP : 30

$$\text{Max } Zp = 5x_1 - 2x_2 + 3x_3$$

s.t. $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 2x_3 \leq 5$

and $x_1, x_2, x_3 \geq 0$.

(b) (i) State the laws of static and kinetic friction

A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place ? 15

(ii) A uniform bar AB, 10ft long, is hinged at B and supported in a vertical plane by a light string AC which connects A to a points C, 10ft directly above B. If AB weights 20lbs and AC = 15ft, find the tension in AC and the reaction at B. 15

7. (a) Solve the following transportation problem : 30

Destinations

Origins		D_1	D_2	D_3	Supply
	O_1	2	7	4	5
	O_2	3	3	1	8
	O_3	5	4	7	7
	O_4	1	6	2	14
	Demand	7	9	18	34

(b) (i) State and prove the principle of angular momentum and the principle of energy for a particle of mass m moving with velocity $\vec{v}$. 20

(ii) State and prove D'Alembert's principle. 10

8. (a) A company has 4 machines to do 3 jobs. Each job can be assigned to one and only

one machine. The cost of each job on each machine is given in the following table : 30

Machines

		W	X	Y	Z
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

What are the job assignments which will minimize the cost ?

(b) (i) A ball is thrown with an initial velocity of 90 ft/sec to strike an object at a horizontal distance of 120 ft and at a height of 40 ft above the ground. At what angle to the horizontal should the ball be thrown ?(Neglect air resistance and the height above ground of the point of projection). 20

(ii) Mud is thrown off from the tire of a wheel (radius a) of a car travelling at a speed v, where $v^2 > ga$. Neglecting the resistance of the air, show that no mud can rise higher than a height $a + v^2/2g + ga^2/2v^2$ above the ground. 10