MPSC

Forest Service
(Mains)

Previous Year Paper
(Mathematics)
13 May, 2025

महाराष्ट्र वन सेवा मुख्य परीक्षा - 2024 दिनांक - 93 मे, 2024

2024

C20

BOOKLET NO.
260026

Forest Services

Mathematics

Time Allowed : Three Hours **Maximum Marks : 200**

Medium : English **Type of Paper : Conventional**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

1. There are **EIGHT** questions divided in two Sections, out of which **FIVE** are to be attempted.
2. Questions No. 1 and 5 are compulsory. Out of the remaining questions, THREE are to be attempted choosing at least ONE question from each Section.
3. The number of marks carried by a question/sub question is indicated against it.
4. Keep in mind the word limit indicated in the question if any.
5. Wherever option has been given, only the required number of responses in the serial order attempted shall be assessed. Unless struck off, attempt of a question shall be counted even if attempted partly. Excess responses shall not be assessed and shall be ignored.
6. Candidates are expected to answer all the sub-questions of a question together. If sub-question of a question is attempted elsewhere (after leaving a few page or after attempting another question) the later sub-question shall be overlooked.
7. Any page or portion of the page left blank in the Answer Booklet must be clearly struck off.
8. Unless otherwise mentioned, symbol and notation have their usual standard meanings. Assume suitable data, if necessary and indicate the same clearly.
9. Neat sketches may be drawn, wherever required.
10. The medium of answer should be mentioned on the answer book as claimed in the application and printed on admission card. The answers written in medium other than the authorized medium will not be assessed and no marks will be assigned to them.

Note : Candidates will be allowed to use Scientific (Non-programmable type) calculators.

SEAL

P.T.O.

SECTION – A

Q1. Solve **any five** out of seven. **(8×5=40)**

(a) Define a differentiable of a function 'f' at a point 'a' in a domain $D \subset R^n$. Prove that every differentiable function is continuous at the point.

(b) Define normal subgroup N of a group G and also define quotient group G/N. Prove that a mapping $\phi : G \to G/N$ given, by $\phi(x) = N_x, \forall x \in G$, is homomorphism.

(c) If f(z) = u (x, y) + iv (x, y) is differentiable at any point z = x + iy in domain D, then prove that its real part u (x, y) and imaginary part v (x, y) are also differentiable at (x, y) and $u_x = v_y$ and $u_y = -v_x$.

(d) Let x = a cosnt + b sinnt be the position of a particle moving in a straight line. Prove that it executes Simple Harmonic Motion (SHM) of period $2\pi/n$ and amplitude $\sqrt{a^2 + b^2}$.

(e) Prove that a set A is closed if and only if its complement B is open in real number system R.

(f) Show that the power series $\sum a_n Z^n$ is

i) either convergent $\forall$ Z

ii) Or converges for Z = 0 only

iii) Or converges for Z in some region of complex plane.

(g) For three vectors a, b, c, prove that a × (b × c) = (a.c) b – (a.b) c.

Q2. (a) i) Find the volume enclosed by the surfaces

$x^2 + y^2 = cz, x^2 + y^2 = 2ax, z = 0.$ **5**

ii) Evaluate $\int_0^1 \frac{\sin^{-1} x}{x} dx$. **10**

(b) State and prove Cayley's theorem. **15**

(c) Let f be vector point differentiable function in a region of space, then for a vector r = xi + yj + zk, prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$. Hence find f(r) such that $\nabla^2 f(r) = 0$. **10**

Q3. (a) State and prove Cauchy's Residue theorem. **15**

(b) Prove that the general equation of the second degree in x and y. **15**

$\phi(x, y) = ax^2 + 2hxy + by^2 + 2gx + zfy + c = 0$ represents a conic section.

(c) Find the equation of a line through a point (x, y, z) and having the direction cosines l, m, n. **10**

Q4. (a) Define Gamma function $\Gamma(n)$ of n and Beta function B (m, n) of m and n.

Prove that $B(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. **15**

(b) Discuss the Newton-Raphson method to find the root of an equation. Further, using it find the root of the equation $x^3 - 2x - 5 = 0$. **15**

(C) If $f = (a + r)\, r^n$, show that divf = 0 and curl $f = (n + 2)r^n a - nr^{n-2}(a.r).r$. **10**

P.T.O.

SECTION – B

Q5. Solve **any five** out of seven. **(8×5=40)**

(a) Define Hermitian and Skew Hermitian matrix with example. If A is square matrix prove that $A + A^*$ is Hermitian and $A - A^*$ is Skew Hermitian (where A^* is transpose conjugate).

(b) How partial differential equations are formed ? Form the partial differential equation given $f(x^2 + y^2, z - xy) = 0$.

(c) Using Simpson's $\frac{1}{3}^{rd}$ rule, estimate approximately the area of cross section of a river 80 feet wide, the depth (d) (in feet) at a distance x from one bank being given by following table.

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

(d) State important features of hexadecimal number system. Convert 2003.31 into its equivalent hexadecimal number (upto 4 digits after decimal point).

(e) Show that the equation $(px - y)(px - 2y) + x^3 = 0$ may be reduced to Clairaut's form by substituting $y = vx$ and hence find its general and singular solution.

(f) A bead slides on a smooth rod which is rotating about one end in a vertical plane with uniform angular velocity ω. Find the equation of motion.

(g) Using Trapezoidal rule, calculate the value of integral $\int_4^{5.2} \log_e x\,dx$ given.

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log_e x$	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486

Compare it with the exact value.

Q6. (a) Find non-singular matrices P and Q such that the matrix A is reduced to normal form. Also find its rank, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$ **15**

(b) Solve $xp + yq = pq$. **15**

(c) For a = 53, b = 28 calculate

i) bitwise AND (a & b)

ii) bitwise XOR ($a \wedge b$). **10**

Q7. (a) Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$ for x = 2 by using Runge Kutta method with initial values x = 0, y = 1 and interval length 0.5. **15**

(b) A salesman has to visit five cities P, Q, R, S and T. The intercity distances are tabulated below :

From/to	**P**	**Q**	**R**	**S**	**T**
P	–	12	24	25	15
Q	6	–	16	18	7
R	10	11	–	18	12
S	14	17	22	–	16
T	12	13	23	25	–

With restriction, if the salesman starts from city P and has to come back to city P, which route would you advice him to take so that total distance travelled by him is minimized ? **15**

(c) Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$. **10**

Q8. (a) Prove that the necessary and sufficient condition for differential equation $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Hence solve, the differential equation $(2y^2 + 3xy - 2y + 6x)dx + (x + 2y - 1)xdy = 0$. **15**

(b) Write Hamilton's equations in polar co-ordinates. **15**

(c) A company manufactures two types of Bags S : small and B : Big. The raw material and labour available per day are 60 units and 50 hours respectively. S requires 2 units of raw material and 5 hours of labour, whereas B requires 6 units of raw material and 2 hours of labour. It is observed that, however they try, the total number of bags produced per day, does not exceed 12. Solve the problem for maximising profit, if the profit for small bag is ₹ 50 and for big one is ₹ 100. **10**

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