Hall Ticket Number

Q.B. No. 100045

Booklet Code : A

Marks : 100

JL-413-MAT

Time : 120 Minutes

Paper-III

Signature of the Candidate

Signature of the Invigilator

INSTRUCTIONS TO THE CANDIDATE
(Read the Instructions carefully before Answering)

1. Separate Optical Mark Reader (OMR) Answer Sheet is supplied to you along with Question Paper Booklet. Please read and follow the instructions on the OMR Answer Sheet for marking the responses and the required data.
2. The candidate should ensure that the **Booklet Code printed on OMR Answer Sheet and Booklet Code supplied are same.**
3. **Immediately on opening the Question Paper Booklet by tearing off the paper seal, please check for *(i)* The same booklet code (A/B/C/D) on each page. *(ii)* Serial Number of the questions (1-100), *(iii)* The number of pages and *(iv)* Correct Printing.** In case of any defect, please report to the invigilator and ask for replacement of booklet with same code within five minutes from the commencement of the test.
4. Electronic gadgets like Cell Phone, Calculator, Watches and Mathematical/Log Tables are not permitted into the examination hall.
5. **There will be 1/4 negative mark for every wrong answer.** However, if the response to the question is left blank without answering, there will be no penalty of negative mark for that question.
6. Record your answer on the OMR answer sheet by using Blue/Black ball point pen to darken the appropriate circles of (1), (2), (3) or (4) corresponding to the concerned question number in the OMR answer sheet. Darkening of more than one circle against any question automatically gets invalidated and will be treated as wrong answer.
7. Change of an answer is **NOT** allowed.
8. Rough work should be done only in the space provided in the Question Paper Booklet.
9. **Return the OMR Answer Sheet and Question Paper Booklet to the invigilator before leaving the examination hall.** Failure to return the OMR sheet and Question Paper Booklet is liable for criminal action.

This Booklet consists of **17** Pages for **100** Questions **+2** pages of Rough Work **+1** Title Page i.e. Total **20** pages

1. If $\{a_n\}$ is a bounded sequence of real numbers, then the sequence $\left\{\frac{a_n}{n^3}\right\}$ is :

(1) divergent

(2) converges to one

(3) converges to zero

(4) a Cauchy sequence but not convergent

2. The series $\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)}{n^2}$ is :

(1) absolutely convergent

(2) conditionally convergent

(3) converges but not conditionally

(4) divergent

3. If $[x]$ and $\{x\}$ denote the greatest integer value and fractional value of x respectively, then $f(x) = [x]^2 - \{x\}^2$ is :

(1) continuous on $[-1\ 1]$

(2) continuous on $[-1\ 1)$

(3) continuous on $(-1\ 1]$

(4) continuous on $(-1\ 1)$

4. Let $f : \mathbf{R} \to \mathbf{R}$ defined as :

$$f(x) = \begin{cases} \frac{1}{n}, & \text{if } x = \frac{m}{n} \text{ is rational and g.c.d. of } (m, n) = 1 \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Then $f(x)$ is :

(1) continuous at $x = 1$ but not at $x = \sqrt{3}$

(2) continuous at $x = 1$ and $x = \sqrt{3}$

(3) continuous at $x = \sqrt{3}$ but not at $x = 1$

(4) discontinuous at $x = 1$ and $x = \sqrt{3}$

5. The sequence of functions $\{f_n(x)\}$ is defined by $f_n(x) = \frac{x}{n}$. Then $\{f_n(x)\}$:

(1) converges uniformly to zero on **R**

(2) converges pointwise to zero but not uniformly on $[0, 1]$

(3) converges uniformly to zero on $[0, 1]$

(4) does not converge on $[0, 1]$

6. Define $f : [0, 1] \to \mathbf{R}$ by

$$f(x) = \begin{cases} 2, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Then :

(1) f is Riemann integrable and $\overline{\int_0^1} f(x)dx = \underline{\int_0^1} f(x)dx = 2$

(2) f is not Riemann integrable and $\overline{\int_0^1} f(x)dx = 2, \underline{\int_0^1} f(x)dx = 0$

(3) f is Riemann integrable and $\overline{\int_0^1} f(x)dx = \underline{\int_0^1} f(x)dx = 0$

(4) f is not Riemann integrable and $\overline{\int_0^1} f(x)dx = 0, \underline{\int_0^1} f(x)dx = 2$

7. If k and l are respectively the supremum and infimum of the set $E = \left\{ \frac{(-1)^n}{n^2} \middle/ n \in N \right\}$, then length of the interval $[l, k]$ is :

(1) 1 (2) $\frac{1}{4}$

(3) $\frac{5}{4}$ (4) $\frac{3}{4}$

8. $f : [0, 1] \to \mathbf{R}$ defined as

$$f(x) = \begin{cases} x^2 \sin \dfrac{\pi}{2x^2}, & x \in (0, 1] \\ 0, & x = 0 \end{cases}$$

is :

(1) discontinuous at $x = 0$

(2) continuous but not differentiable on $[0, 1]$

(3) differentiable on $[0, 1]$ but not of bounded variation

(4) bounded variation on $[0, 1]$ and differentiable

9. $\{S_n\}$ is a sequence of real numbers such that $\{S_{n+1} - S_n\} < \dfrac{1}{2^n}$ for all $n \in N$. Then $\{S_n\}$ is :

(1) a Cauchy Sequence and is divergent

(2) a Cauchy Sequence

(3) monotonically increasing and not bounded

(4) monotonically decreasing

10. $f : [1, 2] \to \mathbf{R}$ and $g : [1, 2] \to \mathbf{R}$ are defined by $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{\sqrt{x}}$. Consider the following statements :

(*a*) The slope of the tangent to the curve $y = f(x)$ parallel to the line joining $(1, 1)$ and $\left(2, \sqrt{2}\right)$ is at $\dfrac{1}{\sqrt{2}} + h(h > 0)$

(*b*) The slope of the tangent to the curve $y = g(x)$ parallel to the line joining $(1, 1)$ and $\left(2, \dfrac{1}{\sqrt{2}}\right)$ is at c where $c^3 = \sqrt{2} + 2h(h > 0)$

Which of the above statement(s) is(are) true ?

(1) Only (*a*) is true

(2) Only (*b*) is true

(3) Both (*a*) and (*b*) are false

(4) Both (*a*) and (*b*) are true

11. $f : [-1, 1] \to \mathbf{R}$ defined by

$$f(x) = \begin{cases} x^{P} \sin\left(x^{-a}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

where P and a are real numbers, $a > 0$. Consider the series $\Sigma u_n = \sum_{n=1}^{\infty} \frac{1}{n^{p}}$.

Then :

(1) $f'(x)$ is bounded for every P and Σu_n is divergent

(2) $f'(x)$ is continuous for every P and Σu_n is divergent

(3) $f'(0)$ exists if Σu_n is convergent

(4) $f(x)$ is continuous for every P

12. $t_n = (-1)^n \left(1 + \frac{1}{n}\right)$, then $\limsup_{n \to \infty} t_n$ is :

(1) -1 (2) 0

(3) does not exist (4) 1

13. Which of the following is a connected subset of $\mathbf{R}$, where a, b, c are real numbers and $a < b < c$?

(1) $\mathbf{Z}$ (2) $\mathbf{Q}$

(3) $[a, b)$ (4) $[a, b) \cup (b, c]$

14. The real line $\mathbf{R}$ with the metric

$$d(x, y) = \begin{cases} 4, & \text{if } x \neq y \\ 0, & \text{if } x = y \end{cases}$$

is :

(1) complete (2) separable

(3) compact (4) connected

15. If $M_n(\mathbf{R})$ denotes the metric space of all $n \times n$ square matrices with real entries, the metric induced by the norm $\|A\| = \left(\sum_{ij} |a_{ij}|^2\right)^{\frac{1}{2}}$ where $A = [a_{ij}]_{n \times n}$ and if $S_n(\mathbf{R})$ and $T_n(\mathbf{R})$ denote the sets of singular and non-singular matrices respectively, then :

(1) both $S_n(\mathbf{R})$ and $T_n(\mathbf{R})$ are open

(2) $S_n(\mathbf{R})$ is open and $T_n(\mathbf{R})$ is closed

(3) $S_n(\mathbf{R})$ is closed and $T_n(\mathbf{R})$ is open

(4) Both $S_n(\mathbf{R})$ and $T_n(\mathbf{R})$ are closed

16. Which of the following is *not* a normal space ?

(1) **R** with usual topology

(2) **R** with discrete topology

(3) $\mathbf{R}_L$, (**R** with Lower limit topology)

(4) $\mathbf{R}_L \times \mathbf{R}_L$

17. Which of the following subsets of $\mathbf{R} \times \mathbf{R}$ is connected ?

(1) $\left\{(x, y)/x^2 + y^2 = 1\right\}$

(2) $\left\{(x, y)/x^2 - y^2 - 1\right\}$

(3) $\left\{(x, y)/xy \neq 0\right\}$

(4) $\left\{(x, y)/x \notin \mathbf{Q}, y \notin \mathbf{Q}\right\}$

18. If A is any connected subset of an infinite metric space (X, d) with at least two distinct points, then A is :

(1) a set with exactly two points

(2) a finite set with at least two points

(3) a countably infinite set

(4) an uncountable set

19. If the function $f : \mathbf{Q} \to \mathbf{Q}$ is defined by

$$f(x) = \begin{cases} -1, & \text{if } x^2 < 2 \\ 1, & \text{otherwise} \end{cases}$$

on the set **Q** of all rational numbers with usual metric, then f is :

(1) Continuous on **Q**

(2) Discontinuous at $x = \sqrt{2}$

(3) Darboux continuous

(4) Continuous but not differentiable

20. The set $x = \{(x, y) \in \mathbf{R} \times \mathbf{R}/x > 0\}$ with the co-finite topology is :

(1) both second countable and separable

(2) separable but not second countable

(3) neither second countable nor separable

(4) second countable but not separable

21. In the topological space $\mathbf{Q}$ of rational numbers with usual topology, the set $E = \left(-\sqrt{2}, \sqrt{2}\right) \cap \mathbf{Q}$ is :

(1) compact but not closed
(2) closed but not bounded
(3) closed and bounded but not compact
(4) compact, closed and bounded

22. If ϕ denotes the Euler's phi function, then $\phi(1000)$ =

(1) 500 (2) 400
(3) 100 (4) 40

23. If $n^{10} + 1$ is divisible by 10, then a possible value of n from the following is :

(1) 10 (2) 11
(3) 12 (4) 13

24. If n is an even number with $n > 6$, then there exist two primes p and q such that :

(1) g.c.d. $(np, nq) = 1$ (2) g.c.d. $(n - p, n - q) = 1$
(3) g.c.d. $(n^p, n^q) = 1$ (4) g.c.d. $(n^2p, n^2q) = 1$

25. The equation $25x \equiv 4$ (modulo 11) has :

(1) infinitely many solutions for x modulo 11
(2) only two solutions for x modulo 11
(3) only one solution for x modulo 11
(4) no solution for x modulo 11

26. If $\bar{a}$ and $\bar{b}$ denote residue classes modulo p and if $\bar{a} = \bar{b}$, then :

(1) p divides ab (2) p divides $a + b$
(3) p divides $a - b$ (4) p divides $\frac{a}{b}$

27. If $a \equiv b \pmod{k}$ and $0 \le |a - b| < k$, then :

(1) g.c.d. $(a, b) = 1$ (2) $a > b$
(3) $a < b$ (4) $a = b$

28. If P is prime, which of the following is true ?

(1) $(P - 1)! \equiv -1 \pmod{P}$ (2) $P! \equiv 1 \pmod{P}$
(3) $(P - 1)! \equiv 1 \pmod{P}$ (4) $P! \equiv -1 \pmod{P}$

29. Which of the following is *not* true ?

(1) $(12)^P \equiv 12 \pmod{P}$ (P is Prime)
(2) $\sum_{d/125} \phi(d) = 125$ (ϕ is Euler's phi function)
(3) $12x \equiv 48 \pmod{18}$ has no solution
(4) $r^5 + 1$ is divisible by 5, $0 \le r \le 9 \Rightarrow r = 4, 9$

30. If ϕ is the Euler totient function, then $\sum_{d/125} \phi(d) =$

(1) 125 (2) 25
(3) 115 (4) 5

31. Let a be a non-zero element in a group G with $O(a) = n$ and m is relatively prime to n. Then :

(1) $O(a^n) < m$ (2) $O(a^m) = n^m$

(3) $O(a^m) < n$ (4) $O(a^m) = n$

32. If [A : B] denote the index number of A in B, and H, K are two subgroups of a finite group G such that $H \subseteq K$, then :

(1) $[G : H] = [G : K][G : H]$ (2) $[G : H] = [G : K][K : H]$

(3) $H \cup K$ is not a subgroup (4) $[G : H \cup K] = [G : H]$

33. If G is a finite non abelian group of order 27 and if Z(G) is center of G, then :

(1) $Z(G) = \{e\}$ (2) $Z(G) = G$

(3) $O(Z(G)) = 3$ (4) $O(Z(G)) = 9$

34. If G is a finite group and $O(G) = 28$, then the number of 7-Sylow subgroups of G are :

(1) two (2) one

(3) three (4) infinite

35. In the group $Z_4 \times Z_4$, the number of subgroups of order 4 is :

(1) 16 (2) 8

(3) 6 (4) 1

36. A subgroup of order 9 of the group $Z_3 \times Z_{15}$ is :

(1) Z_9 (2) $Z_3 \times Z_3$

(3) $Z_3 \times Z_6$ (4) Z_5

37. In a group of order 4, if $a = a^{-1} \; \forall \; a \in G$, then number of subgroups of G is :

(1) 2 (2) 3

(3) 4 (4) 5

38. If (Z, +) is a group, then $O\left(\frac{Z}{4Z}\right)$ is :

(1) infinite (2) 1

(3) 4 (4) 8

39. If order of a group is 231, then the number of elements of order 11 in that group is :

(1) 0 (2) 21

(3) 11 (4) 10

40. **Q** and **R** are the rings of rational and real numbers with respect to usual addition and multiplication. $S = \{(x, y, 0)/x \in \mathbf{R}, y \in \mathbf{Q}\}$ is :

(1) an integral domain but not a field

(2) field

(3) commutative ring with unity with zero divisors

(4) non-commutative ring without zero divisors

41. The number of non-zero nilpotent elements in an integral domain is :

(1) 0

(2) 1

(3) 2

(4) the order of the integral domain

42. The number of ideals of order 25 in the ring Z_{100} is :

(1) 5 (2) 4

(3) 2 (4) 1

43. The number of prime ideals in the ring (**Q**, +, .) of rational numbers is :

(1) 0 (2) 2

(3) infinite (4) 1

44. If **Z** denotes the ring of integers, then the number of non-zero ring homomorphisms from **Z** to **Z** is :

(1) 1 (2) 2

(3) 3 (4) 5

45. The characteristic of a Boolean ring is :

(1) 0 (2) 1

(3) 2 (4) 4

46. In Z_6, the number of idempotent elements and nilpotent elements are denoted by x and y respectively, then :

(1) $x < y$ (2) $x > y$

(3) $x = y$ (4) $x + y = 8$

47. If Z_n denotes the ring of integers modulo n and $\psi : Z \to Z_2 \times Z_3$ defined by $\psi(n) = (\hat{n}, \hat{n})$, then the kernel of ψ is :

(1) $\{0\}$ (2) 6Z

(3) 3Z (4) 2Z

48. In the ring of Gaussian integers if U is an ideal, then it is a :

(1) prime ideal (2) principal ideal

(3) maximal ideal (4) a field

49. The dimension of a vector space of polynomials of degree $\leq n$ over a field F is :

(1) $n + 1$ (2) n

(3) $n - 1$ (4) infinity

50. A subspace of the vector space $V_3(\mathbf{R})$ among the following is :

(1) $\{(x, y, z)/xy < 0\}$ (2) $\{(x, y, z)/x < 0\}$

(3) $\{(x, y, z)/x^2 + y^2 + z^2 \leq 1\}$ (4) $\{(x, y, z)/x + z = 0\}$

51. A Vector in $V_2(\mathbf{R})$, which is not in the linear span of $S = \{(1, 2), (3, 6)\} \subseteq V_2(\mathbf{R})$, is :

(1) (5, 10) (2) (–3, –6)

(3) (4, 7) (4) (4, 8)

52. A basis of $\mathbf{R}^3(\mathbf{R})$ is :

(1) {(3, 0, 0), (0, 4, 0), (1, 1, 0)} (2) {(7, 0, 0), (0, 7, 0), (7, 7, 7)}

(3) {(5, 0, 0), (0, 0, 0), (2, 1, 1)} (4) {(1, 0, 0), (5, 0, 0), (0, 0, 2)}

53. The determinant of the matrix of the linear Transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by $T(a, b, c) = (3a + b, -2a + b, -a + 2b + 4c)$ with respect to any basis is :

(1) 10 (2) 6

(3) 2 (4) 20

54. If S and T are subsets of a Vector Space V(F), then $L(S \cup T) =$

(1) L(S) (2) L(T)

(3) L(S) + L(T) (4) $L(S) \cup L(T)$

55. Let $V_2(\mathbf{C})$ be the inner product space with respect to the standard inner product. A Vector in $V_2(\mathbf{C})$ orthogonal to the Vector $(1 - i, 1 + i)$ is :

(1) $(1 + i, 1 + i)$ (2) $(-1 + i, 1 + i)$

(3) $(1 + i, 1 - i)$ (4) $(-1 - i, 1 - i)$

56. In an inner product space V(**F**) :

(1) $|(\alpha, \beta)| \leq \|\alpha\| + \|\beta\|$ (2) $\|(\alpha, \beta)\| \leq \|\alpha\| - \|\beta\|$

(3) $|(\alpha, \beta)| \leq \|\alpha\| \, \|\beta\|$ (4) $\|(\alpha, \beta)\| \leq \|\alpha\| \, \|\beta\|$

57. The eigenvalues of a 3×3 matrix P are 3, 1 and –2, then $6P^{-1} =$

(1) $5I - 2P + P^2$ (2) $5I + 2P + P^2$

(3) $5I + 2P - P^2$ (4) $5I - 2P - P^2$

58. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then $A^{50} =$

(1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 0 & 1 \\ 25 & 1 & 0 \end{bmatrix}$

59. A linear transformation $T : F^2 \to F^3$ is defined as $f(x, y) = (x, x + y, y)$. Then the nullity of T is :

(1) 4
(2) 3
(3) 0
(4) 2

60. The possible set of eigenvalues of an orthogonal skew-symmetric matrix of order 4×4 is :

(1) $\{0, i, -i\}$
(2) $\{1, -1, i, -i\}$
(3) $\{1, -1\}$
(4) $\{i, -i\}$

61. The trace and determinant of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are respectively 1 and -3. The trace of $A^4 - A^3$ is :

(1) 21
(2) 9
(3) 4
(4) 0

62. If an eigenvector of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, then its corresponding eigenvalue is :

(1) 6
(2) 1
(3) -2
(4) -3

63. $f : \mathbf{R}^3 \to \mathbf{R}^3$ is a linear transformation defined by

$f(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3, x_3 - x_1)$. If $(a, b, c) \in$ Ker f, then :

(1) $a + b + c = 0$ (2) $a \neq b = c$

(3) $a = b = c$ (4) $a = b \neq c$

64. Let A = $\begin{bmatrix} 1 & \alpha & 0 \\ \alpha & 1 & 0 \\ 0 & 1 & \alpha \end{bmatrix}$. Consider the following statements :

(*a*) Rank of A is maximum only when a $\alpha \neq 0$

(*b*) Rank of A is one when $\alpha = 0$ or 1 or -1

(c) Rank of A is two only when $\alpha = 0$

Which of the above statements are *not* correct ?

(1) (*a*), (*c*) only (2) (*a*), (*b*) only

(3) (*a*), (*b*) and (*c*) (4) (*b*), (c) only

65. 1, ω, ω^2 are cube roots of unity. Each of α, β, γ is either ω or ω^2. If the rank of the matrix $\begin{bmatrix} 1 & \alpha & \beta \\ \omega & 1 & \gamma \\ \omega^2 & \omega & 1 \end{bmatrix}$ is three, then one of the possible triplet (α, β, γ) is :

(1) $(\omega, \omega, \omega^2)$ (2) $(\omega, \omega^2, \omega)$

(3) $(\omega^2, \omega^2, \omega^2)$ (4) $(\omega^2, \omega^2, \omega)$

66. Let $f : \mathbf{C} \to \mathbf{C}$ be analytic except for a simple pole at $z = 0$ and $g : \mathbf{C} \to \mathbf{C}$ be analytic on C. Then the value of $\dfrac{\operatorname{Res}\{f(z)g(z)\}\, at z = 0}{\operatorname{Res} f(z)\, at z = 0}$ is :

(1) $f(0)$ (2) $g'(0)$

(3) $f'(0)$ (4) $g(0)$

67. Let F(z) be an entire function on **C** such that $|F(z)| \le 100$ for each z with $|z| \ge 2$. If $F(i) = 2i$, then F(1) is :

(1) any real number (2) $2i$

(3) 2 (4) 0

68. The transformation $w = z^2$ transforms the lines $x = 0$, $y = 0$ and $x + y = 1$ into the curves c_1, c_2 and c_3 respectively. Then at $w = 0$ the angle between c_1 and c_2 is :

(1) $\frac{\pi}{2}$ (2) π

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

69. The radius of convergence of the series $\sum_{k=1}^{\infty} \frac{z^{2k+1}}{(2k+1)!}$ is :

(1) infinity (2) 1

(3) e (4) $\frac{1}{e}$

70. The coefficient of $(z - \pi)^2$ in the Taylor's Series expansion around π of

$$f(z) = \begin{cases} \frac{\sin z}{z - \pi}, & \text{if } z \ne \pi \\ -1, & \text{if } z = \pi \end{cases}$$

is :

(1) $\frac{1}{6}$ (2) $\frac{-1}{2}$

(3) $\frac{1}{2}$ (4) 0

71. Let $f : \mathbf{C} - \{3i\} \to \mathbf{C}$ be defined as $f(z) = \frac{z - i}{iz + 3}$. Which of the following is false ?

(1) All the fixed points of f are in the region $Im(z) > 0$

(2) There is no straight line which is mapped onto a straight line by f

(3) f is conformal

(4) f maps circles onto circles

72. The power series $\sum_{n=1}^{\infty} z^n$ is analytic on :

(1) $\{z \in \mathbf{C} / |z| < 1\}$

(2) $\{z \in \mathbf{C} / |z| \leq 1\}$

(3) $\{z \in \mathbf{C} / \frac{1}{2} \leq |z| < 2\}$

(4) nowhere

73. For $f(z) = e^{\frac{1}{z}}$, $z = 0$ is :

(1) a removable singularity

(2) a pole of order 1

(3) an essential singularity

(4) a pole of order 2

74. If $w = f(z) = u + iv$ is an analytic function and $P(\alpha, \beta)$ is a point on the two families of curves $u(x, y) = k$, $v(x, y) = l$ (k and l are constants), then the reciprocal of the slope of tangent at P to $u(x, y) = k$ is :

(1) equal to the slope of the tangent at $P(\alpha, \beta)$ to the curve $v(x, y) = l$

(2) negative of the slope of the tangent at $P(\alpha, \beta)$ to the curve $v(x, y) = l$

(3) reciprocal of the slope of the tangent at $P(\alpha, \beta)$ to the curve $v(x, y) = l$

(4) negative of the slope of the tangent at $P(\alpha, \beta)$ to the curve $u(x, y) = k$

75. $\frac{1}{2\pi i}\int_c \frac{3e^{2z}}{(z-1)^4} dz =$

(1) 0

(2) $2e^{-2}$

(3) $\frac{8\pi i}{3}e^2$

(4) $4e^2$

76. If y_1, y_2 and y_3 are three solutions of $(D^3 + aD^2 + bD + c)y = 0$ and determinant of $\begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix} \neq 0$ then :

(1) $y_1 = ky_2 + ly_3$ (2) $y_1' = 0, y_1'' = 0$

(3) $y_3 = k_1y_1 + l_1y_2$ (4) $y_1 \neq 0, y_2 \neq 0, y_3 \neq 0$

77. The general solution of $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ is $y =$

(1) $Ax + Bx^2$ (2) $Ax + Bx\log x$

(3) $Ax + B\log x$ (4) $A + Bx\log x$

78. The differential equation $\dfrac{d^2z}{dt^2} + \sin(t + z) = \sin t$ is :

(1) non-linear and non-homogeneous

(2) non-linear and homogeneous

(3) linear and homogeneous

(4) linear and non-homogeneous

79. All the zeros of the polynomial $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n$ have negative real parts. If $u(t)$ is any solution of the differential equation

$(a_0D^n + a_1D^{n-1} + a_2D^{n-2} + ... + a_n)u = 0$, where $D = \dfrac{d}{dt}$, then $\lim\limits_{t \to \infty} u(t) =$

(1) a negative real number (2) a positive real number

(3) a non-zero real number (4) zero

80. If $y' \neq x$, a solution of the differential equation $y'(y' + y) = x(x + y)$ is $y =$

(1) $1 - x - e^{-x}$ (2) $1 - x + e^{x}$

(3) $1 + x + e^{-x}$ (4) $1 + x + e^{x}$

81. If $y = x\cos 2x$ is a particular solution of $y'' + ay = -4\sin 2x$, then the constant a takes the value :

(1) -4 (2) 4

(3) -2 (4) 2

82. Which of the following pair of functions is *not* a linearly independent pair of solutions of $y'' + 9y = 0$:

(1) $\sin 3x,\ 5\cos 3x - \sin 3x$

(2) $\cos 3x,\ 3\sin x, - 4\sin^3 x,$

(3) $\sin 3x + \cos 3x,\ -3\cos x + 4\cos^3 x$

(4) $\cos 3x,\ 5\cos^3 x - \frac{15}{4}\cos x$

83. If a transformation $y = uv$ transforms $f(x)y'' - 4f'(x)y' + g(x)y = 0$ to the form $v'' + h(x)v = 0$, then u is equal to :

(1) xf

(2) $\frac{1}{f^2}$

(3) f^2

(4) $\frac{1}{2f}$

84. If $y_1 = e^{2x}$ and $y_2 = xe^{2x}$ are two independent solutions of a differential equation $y'' + Q(x)y' + R(x)y = 0$ and $W(y_1, y_2)$ is the Wronskian of y_1 and y_2, then $[W(y_1, y_2)Q]\ (0) =$

(1) 4

(2) –4

(3) 1

(4) 0

85. If $ye^{xy}dx + (xe^{xy} + 2y)dy = d(f(x, y))$, then $f(x, y) =$

(1) $e^{x+y} + y^2$

(2) $e^{xy} + y^2$

(3) $e^x + e^y + y^2$

(4) $y^2 + e^x - e^{yx}$

86. The general solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is :

(1) $F(x + ct) + G(x - ct)$

(2) $F(x + ct) + G(x + ct)$

(3) $F(x - ct) + F'(x - ct)$

(4) $F(x + ct) + F(x - ct)$

87. The one-dimensional heat equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial t^2}$ is :

(1) elliptic

(2) hyperbolic

(3) parabolic

(4) mixed

88. Solution of the problem $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $t > 0$, $-\infty < x < \infty$ satisfying the conditions $u(x, 0) = x$, $\frac{\partial u}{\partial t}(x, 0) = 0$ is :

(1) x (2) $\frac{x^2}{2}$

(3) $2x$ (4) $2t$

89. When $x < 0$, $\frac{\partial^2 u}{\partial x^2} - x\frac{\partial^2 u}{\partial y^2} = 0$ is :

(1) elliptic (2) parabolic

(3) hyperbolic (4) spherical

90. The particular integral of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 18(x + y)$ is :

(1) $x^3 + y^3$ (2) $3(x^2 + 2xy)$

(3) $x^3 + 2x^2y$ (4) $3(x^3 + 3x^2y)$

91. The complete integral of $pq = 1$ is :

(1) $ax + by = z$ (2) $a^2x + y^2 - az = c$

(3) $a^2x + y + az = c$ (4) $a^2x + y - az = c$

92. The particular integral of $(D^2 - D')z = e^{x + y}$ is $\left(D = \frac{\partial z}{\partial x}, D' = \frac{\partial z}{\partial y}\right)$

(1) $x^2e^{x + y}$ (2) $y^2e^{x + y}$

(3) $xye^{x + y}$ (4) $\frac{x}{2}e^{x + y}$

93. The complete solution of the equation $z = p(x + 2) + q(y + 3)$ is :

(1) $z = x + y$ (2) $z = (x + 2)(y + 3)$

(3) $z = xy + k$ (4) $z = a(x + 2) + b(y + 3)$

94. The direction cosines of the normal to the plane $5x - y + 3z = 27$ are :

(1) $\pm\frac{5}{\sqrt{35}}, \pm\frac{1}{\sqrt{35}}, \mp\frac{3}{\sqrt{35}}$ (2) $\pm\frac{5}{\sqrt{35}}, \mp\frac{1}{\sqrt{35}}, \pm\frac{3}{\sqrt{35}}$

(3) $\mp\frac{5}{\sqrt{35}}, \pm\frac{1}{\sqrt{35}}, \mp\frac{3}{\sqrt{35}}$ (4) $\frac{1}{\sqrt{35}}, 0, 0$

95. The perpendicular distance of the point (1, 1, –1) from the line through the point (–3, –1, 1) whose directional ratios are (1, 1, 1) is :

(1) 8 (2) $\sqrt{5}$

(3) $\sqrt{6}$ (4) $2\sqrt{\frac{14}{3}}$

96. The condition for the lines $x = az + b$, $y = cz + d$ and $x = a_1z + b_1$, $y = c_1z + d_1$ to be perpendicular is :

(1) $aa_1 + bb_1 + 1 = 0$ (2) $aa_1 + cc_1 + 1 = 0$

(3) $aa_1 + bb_1 - 1 = 0$ (4) $ab + a_1b_1 + 1 = 0$

97. If the point of intersection of the line $\frac{x+3}{4} = \frac{y+4}{4} = \frac{z-8}{-5}$ and the sphere $x^2 + y^2 + z^2 + 2x - 10y - 23 = 0$ are $(\alpha_1, \beta_1, \gamma_1)$ and $(\alpha_2, \beta_2, \gamma_2)$, then $(\alpha_1 + \alpha_2) + (\beta_1 + \beta_2) + (\gamma_1 + \gamma_2) =$

(1) 9 (2) 8

(3) 6 (4) 10

98. The equation of reciprocal cone of $5x^2 + 2y^2 + 7z^2 = 0$ is :

(1) $\frac{x^2}{7} + \frac{y^2}{2} + \frac{z^2}{5} = 0$ (2) $\frac{x^2}{2} + \frac{y^2}{7} + \frac{z^2}{5} = 0$

(3) $\frac{x^2}{5} + \frac{y^2}{2} + \frac{z^2}{7} = 0$ (4) $\frac{x^2}{7} + \frac{y^2}{5} + \frac{z^2}{2} = 0$

99. If the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ are at right angles, then $a + b + c =$

(1) 1 (2) –1

(3) 2 (4) 0

100. The vertex of the cone

$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$ is :

(1) (2, 2, 1) (2) (–1, –2, –3)

(3) (1, 2, 3) (4) (1, –2, 3)

Space for Rough Work

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