KEA Assistant Professor

Previous Year Paper

March 2022 (Mathematics)

RAPGFGCK-2021-D4MM

ವಿಷಯ	ವಿಷಯ ಸಂಕೇತ	ಸಮಯ	ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯ ವರ್ಷನ್ ಕೋಡ್	ಕ್ರಮ ಸಂಖ್ಯೆ
ಗಣಿತಶಾಸ್ತ್ರ	19	ಬೆ. 9.00 ರಿಂದ ಮ. 12.00 (3 Hours)	A-1	145201

ಒಟ್ಟು ಅವಧಿ	ಉತ್ತರಿಸಲು ಇರುವ ಗರಿಷ್ಠ ಅವಧಿ	ಗರಿಷ್ಠ ಅಂಕಗಳು	ಒಟ್ಟು ಪ್ರಶ್ನೆಗಳು	ನಿಮ್ಮ ಸಂಖ್ಯೆಯನ್ನು ಬರೆಯಿರಿ				
210 ನಿಮಿಷಗಳು	180 ನಿಮಿಷಗಳು	250	125					

ಮಾಡಿ

1. ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರಿಂದ ಈ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ನಿಮಗೆ **ಬೆ. 8.55** ಆದ ನಂತರ ಕೊಡಲಾಗಿರುತ್ತದೆ.
2. ಅಭ್ಯರ್ಥಿಗಳು ನೊಂದಣಿ ಸಂಖ್ಯೆಯನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆದು ಅದಕ್ಕೆ ಸಂಬಂಧಿಸಿದ ವೃತ್ತಗಳನ್ನು ಸಂಪೂರ್ಣವಾಗಿ ತುಂಬಿದ್ದೀರೆಂದು ಖಾತ್ರಿಪಡಿಸಿಕೊಳ್ಳಿ.
3. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯ ವರ್ಷನ್ ಕೋಡ್ ಅನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆದು ಅದಕ್ಕೆ ಸಂಬಂಧಿಸಿದ ವೃತ್ತಗಳನ್ನು ಸಂಪೂರ್ಣವಾಗಿ ತುಂಬಬೇಕು.
4. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯ ವರ್ಷನ್ ಕೋಡ್ ಮತ್ತು ಕ್ರಮಸಂಖ್ಯೆಯನ್ನು ನಾಮಿನಲ್ ರೋಲ್‌ನಲ್ಲಿ ತಪ್ಪಿಲ್ಲದೆ ಬರೆಯಬೇಕು.
5. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯ ಕೆಳಭಾಗದ ನಿಗದಿತ ಜಾಗದಲ್ಲಿ ಪೂರ್ಣ ಸಹಿ ಮಾಡಬೇಕು.

ಮಾಡಬೇಡಿ

1. **ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಮುದ್ರಿತವಾಗಿರುವ ಟೈಮಿಂಗ್ ಮಾರ್ಕನ್ನು ತಿದ್ದಬಾರದು / ಹಾಳುಮಾಡಬಾರದು / ಅಳಿಸಬಾರದು.**
2. **ಮೂರನೇ ಬೆಲ್ ಬೆ. 9.00 ಕ್ಕೆ ಆಗುತ್ತದೆ. ಅಲ್ಲಿಯವರೆಗೂ,**
   - ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಬಲಭಾಗದಲ್ಲಿರುವ ಸೀಲ್ ಅನ್ನು ತೆಗೆಯಬಾರದು.
   - ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಒಳಗಡೆ ಇರುವ ಪ್ರಶ್ನೆಗಳನ್ನು ನೋಡಲು ಪ್ರಯತ್ನಿಸಬಾರದು ಅಥವಾ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಉತ್ತರಿಸಲು ಪ್ರಾರಂಭಿಸಬಾರದು.

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಮುಖ್ಯ ಸೂಚನೆಗಳು

1. ಪ್ರಶ್ನೆಗಳಲ್ಲಿ ಬಳಸಿರುವ signs and symbols ಗಳನ್ನು, ಬೇರೆ ರೀತಿಯಲ್ಲಿ ಹೇಳದ ಹೊರತು, ನಿಗದಿತ ಪಠ್ಯಪುಸ್ತಕದಲ್ಲಿನ ಅರ್ಥವನ್ನು ಪರಿಗಣಿಸಬೇಕು.
2. ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯಲ್ಲಿ ಒಟ್ಟು **125** ಪ್ರಶ್ನೆಗಳಿದ್ದು, ಪ್ರತಿ ಪ್ರಶ್ನೆಗೂ **4** ಬಹು ಆಯ್ಕೆ ಉತ್ತರಗಳು ಇರುತ್ತವೆ. ಪ್ರತಿ ಪ್ರಶ್ನೆಯ ಕೆಳಗೆ ಕೊಟ್ಟಿರುವ ನಾಲ್ಕು ಬಹು ಆಯ್ಕೆಯ ಉತ್ತರಗಳಲ್ಲಿ ಸರಿಯಾದ ಒಂದು ಉತ್ತರವನ್ನು ಆಯ್ಕೆ ಮಾಡಿ.
3. **ಮೂರನೇ ಬೆಲ್ ಅಂದರೆ ಬೆ. 9.00ರ ನಂತರ** ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಬಲಭಾಗದಲ್ಲಿರುವ ಸೀಲ್ ತೆಗೆದು ಈ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯಲ್ಲಿ ಯಾವುದೇ ಪುಟಗಳು ಮುದ್ರಿತವಾಗಿಲ್ಲದೇ ಇರುವುದು ಕಂಡು ಬಂದಲ್ಲಿ ಅಥವಾ ಹರಿದು ಹೋಗಿದ್ದಲ್ಲಿ ಅಥವಾ ಯಾವುದೇ ಐಟಂಗಳು ಬಿಟ್ಟುಹೋಗಿದೆಯೇ ಎಂಬುದನ್ನು ಖಚಿತಪಡಿಸಿಕೊಂಡು, ಈ ರೀತಿ ಆಗಿದ್ದರೆ ಕೂಡಲೇ ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರಿಂದ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಿಕೊಳ್ಳಿ ನಂತರ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಉತ್ತರಿಸಲು ಪ್ರಾರಂಭಿಸುವುದು.
4. ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗೆ ಅನುಗುಣವಾಗಿರುವ ಸರಿ ಉತ್ತರವನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಅದೇ ಕ್ರಮ ಸಂಖ್ಯೆಯ ಮುಂದೆ ನೀಡಿರುವ ಸಂಬಂಧಿಸಿದ ವೃತ್ತವನ್ನು **ನೀಲಿ ಅಥವಾ ಕಪ್ಪು** ಶಾಯಿಯ ಬಾಲ್ ಪಾಯಿಂಟ್ ಪೆನ್‌ನಿಂದ ಸಂಪೂರ್ಣ ತುಂಬುವುದು.

ಸರಿಯಾದ ಕ್ರಮ CORRECT METHOD	ತಪ್ಪುಕ್ರಮಗಳು WRONG METHODS
	(•) (B) (C) (D) (A) (B) (C) (D) (A) ● ● (D)
(A) ● (C) (D)	(•) (B) (C) (D) (A) ● (C) (D)

5. ಈ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಸ್ಕ್ಯಾನ್ ಮಾಡುವ ಸ್ಕ್ಯಾನರ್ ಬಹಳ ಸೂಕ್ಷ್ಮವಾಗಿದ್ದು ಸಣ್ಣ ಗುರುತನ್ನು ಸಹ ದಾಖಲಿಸುತ್ತದೆ. ಆದ್ದರಿಂದ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಉತ್ತರಿಸುವಾಗ ಎಚ್ಚರಿಕೆ ವಹಿಸಿ.
6. ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯಲ್ಲಿ ಕೊಟ್ಟಿರುವ ಖಾಲಿ ಜಾಗವನ್ನು ರಫ್ ಕೆಲಸಕ್ಕೆ ಉಪಯೋಗಿಸಿ. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಇದಕ್ಕೆ ಉಪಯೋಗಿಸಬೇಡಿ.
7. ಕೊನೆಯ ಬೆಲ್ ಅಂದರೆ **ಮ. 12.00** ಆದ ನಂತರ ಉತ್ತರಿಸುವುದನ್ನು ನಿಲ್ಲಿಸಿ.
8. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರಿಗೆ ಯಥಾಸ್ಥಿತಿಯಲ್ಲಿ ನೀಡಿರಿ.
9. ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರು ಮೇಲ್ಭಾಗದ ಹಾಳೆಯನ್ನು ಪ್ರತ್ಯೇಕಿಸಿ (ಕಚೇರಿ ಪ್ರತಿ) ತನ್ನ ವಶದಲ್ಲಿ ಇಟ್ಟುಕೊಂಡು ತಳಬದಿಯ ಯಥಾಪ್ರತಿಯನ್ನು (Candidate's Copy) ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಕೊಡುತ್ತಾರೆ.

ಸೂಚನೆ: ಕನ್ನಡ ಆವೃತ್ತಿಯ ಪ್ರಶ್ನೆಗಳಲ್ಲಿ ಉತ್ತರಿಸುವ ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಕನ್ನಡದಲ್ಲಿ ಮುದ್ರಿತವಾಗಿರುವ ಪ್ರಶ್ನೆಗಳ ಬಗ್ಗೆ ಏನಾದರೂ ಸಂದೇಹವಿದ್ದಲ್ಲಿ ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಯ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ನೋಡಬಹುದು. ಏನಾದರೂ ವ್ಯತ್ಯಾಸ ಕಂಡುಬಂದಲ್ಲಿ ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಯನ್ನು ಅಂತಿಮ ಎಂದು ಪರಿಗಣಿಸಲಾಗುವುದು.

MATHEMATICS

1. Let $f : R \to R$ be such that f, f ' and f '' are all continuous functions with $f > 0$, $f' > 0$, $f'' > 0$ then

$\lim\limits_{x \to -\infty} \frac{f(x) + f'(x)}{2}$ is

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) 2

⇒ ⇒ $\frac{1+1}{2} =$ ⇒ $\frac{x+1}{2}$ ⇒ e^x, e^x ⇒ $\frac{e^x}{2} = 1$

2. Let $f : [0, 1] \to R$ be continuous function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and $|f(x) - f(y) - (x - y)| \leq \sin|x - y|^2 \ \forall \ x, y \in [0, 1]$. Then $\int_0^1 f(x)\, dx$ is

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

$f(x) = -x$ ⇒ $-\frac{1}{2}$ ⇒ $-\frac{x^2}{2}$

3. Let $a \in \mathbb{R}$ the coefficient of x^{125} in Maclaurin series of $(x + a^3)\, e^x$ is $\frac{28}{124!}$ then value of a is

(A) 10

(B) 15

(C) 24

(D) 14

4. Let $\{a_n\}$ be a sequence of reals such that $a_1 = 3$ and $a_{n+1} = \frac{a_n^2 - 2a_n + 4}{2}$, $n \geq 1$ then which of the following statements is true ?

(A) $\{a_n\}$ is a monotonic sequence

(B) $\{a_n\}$ is a bounded sequence

(C) $\{a_n\}$ does not have finite limit as $n \to \infty$

(D) $\lim\limits_{n \to \infty} a_n = 2$

Space For Rough Work

$l = \frac{l^2 - 2l + 4}{2}$

$2l = l^2 - 2l + 4$

$l^2 - 4l + 4 = 0$

$l^2 - 2l - 2l + 4$

$l(l-2) - 2(l-2) \Rightarrow (l-2)^2$

15

125

$x + a^3 = 0$

$x = -a^3$, e^{a^3}

$0 < x+1 < 2$

$\infty > \frac{1}{x+1} > \frac{1}{2}$

$\frac{x}{2} < \frac{x}{x+1} < \infty$

5. The set $\left\{\frac{x}{x+1} : -1 < x < 1\right\}$ as a subset of R is

(A) Connected and compact

(B) Connected but not compact

(C) Not connected but compact

(D) Neither compact nor open

odd

6. Let f: R\{0} → ℝ defined by $f(x) = x + \frac{1}{x^3}$ · On which of the following interval is f one-one ?

(A) $(-\infty, -1)$

(B) $(0, 1)$

(C) $(0, 2)$

(D) $(0, \infty)$

7. The set $x = \mathbb{R}$ with metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$ is

(A) bounded but not compact

(B) bounded but not complete

(C) complete but not bounded

(D) compact but not complete

8. The value of $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k^2}}$ equals

$\int \frac{1}{\sqrt{1+x^2}} = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

(A) $\log\left(1+\frac{1}{\sqrt{2}}\right)$

(B) $\log\left(1+\sqrt{2}\right)$

(C) $\log\left(1-\sqrt{2}\right)$

(D) $\log\left(1-\frac{1}{\sqrt{2}}\right)$

9. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences of real numbers such that the series $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ a converges. Then the series $\sum_{n=1}^{\infty} a_n b_n$

(A) is absolutely convergent

(B) may not converge

(C) is always convergent but not converge absolutely

(D) converges to 0

Space For Rough Work

$\Rightarrow \int_0^\infty \frac{1}{\sqrt{1+x^2}} \Rightarrow \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)$

$\Rightarrow \frac{1}{2}\log\left(1+\sqrt{2}\right)$

$5 \Rightarrow 2\log\left(\frac{1}{1+\sqrt{2}}\right)$

10. Let $a_n = \sum_{k=1}^{n} \frac{n}{n^2 + k}$ for $n \in \mathbb{N}$, then the sequence $\{a_n\}$ is

(A) convergent

(B) bounded but not convergent

(C) diverges to ∞

(D) Neither bounded nor diverges to ∞

11. Choose the correct statement :

(A) Every subset of $\mathbb{R}$ is Lebesgue measurable

(B) Every Lebesgue measurable set is also a Borel set.

(C) $\exists$ a Borel set which is not Lebesgue measurable.

(D) $\exists$ a Lebesgue measurable set which is not a Borel set.

12. Let E be a compact subset of $\mathbb{R}'$. Then

(A) E is also a G_δ set

(B) E cannot be a F_σ set

(C) Interior of E is always non-empty.

(D) There exist real continuous functions on E that are unbounded.

13. For $r \in \mathbb{R}^+$, denote by

$S_r = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = r^2\}$ and let $E = \bigcup_{r \in \mathbb{R}^+ - \otimes} S_r$. Then

(A) E is path-connected

(B) The Lebesgue measure of E is infinite

(C) E contains a non-empty open set.

(D) Every open set containing E^c has infinite Lebesgue measure

14. If $f^+_{(x)} = \max\{f(x), 0\}$ and $f^-(x) = \max\{-f(x), 0\}$, then

(A) $f = f^+ + f^-$

(B) $f = f^-$

(C) $f = f^+ - f^-$

(D) $f = f^+$

Space For Rough Work

15. Fatou's Lemma states that : If $\{f_n\}$ is a sequence of non-negative measurable functions and $f_n(x) \to f(x)$ a.e. on a set E, then

(A) $\int_E f \leq \underline{\lim} \int_E f_n$

(B) $\int_E f \leq \int_E f_n$

(C) $\int_E f \geq \underline{\lim} \int_E f_n$

(D) None of these

16. Which of the following statements is true ?

(A) If $p < 1$, then $S = \{x : \|x\|_p \leq 1\}$ is converse.

(B) If $p < 1$, then $S = \{x : \|x\|_p \leq 1\}$ is not converse.

(C) If $p < 1$, then $\|x\|_p = (|x_1|^p + |x_2|^p)^{1/p}$ on R^2 yields a norm.

(D) If $p = 1$ as in $(c)_3$ then norm defined on R^2 is $\|x\|_1 = |x_1|^2 + |x_2|^2$

17. The space B(N, N') is a Banach space if

(A) N is a Banach space

(B) N' is a Banach space

(C) Either N or N' is a Banach space

(D) The scalar field is R

18. Choose the incorrect statement of the following :

(A) l_1 is separable

(B) l_1^* is separable

(C) The normed linear space N is separable if N* is separable

(D) N* need not be separable whenever the normed linear space N is separable

Space For Rough Work

19. The conjugate space of C_o is

(A) l_∞ (B) l_1

(C) l_∞^2 (D) l'_p

20. Which of the following is true ?

(A) If a normed linear space N is reflexive then it is separable.

(B) If a normed linear space N is reflexive then it is not necessarily complete.

(C) If a normed linear space N is complete then it is reflexive.

(D) If a normed linear space N is not complete, then it is reflexive.

21. Let T be an operator on a normed linear space N, then $T^{**} = T$ if

(A) N is a Banach space (B) N is reflexive

(C) N* is reflexive (D) N* is a Banach space

22. Choose the incorrect statement from the following :

(A) Addition is jointly continuous in a normed linear space.

(B) Scalar multiplication is jointly continuous in a normed linear space.

(C) The inner product is jointly continuous in a normed linear space.

(D) Parallelogram law is true in l_1^n $(n > 1)$

23. The Hilbert cube is a subset of l_2 consisting of all sequences $x = \{x_1, x_2, \ldots\ldots, x_n, \ldots$ satisfying the condition that

(A) $|x_n| \geq n$ for all n (B) $|x_n| \leq n$ for all n

(C) $|x_n| \leq \frac{1}{n}$ for all n (D) $|x_n| \geq \frac{1}{n}$ for all n

Space For Rough Work

24. Choose the correct option.

(A) The set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} forms a vector space over the field {0, 2, 4, 6, 8} under addition and multiplication (mod 10).

(B) The set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13} does not form a vector space over the field {0, 2, 4, 6, 8, 10, 12} under addition and multiplication (mod 14).

(C) Set of all 2×2 matrices with real entries forms a vector space over the field of matrices F with $F = \left\{ \begin{bmatrix} r & r \\ r & r \end{bmatrix} \middle| r \in R \right\}$ under usual addition and multiplication of matrices.

(D) Set of all 2×2 matrices with real entries does not form a vector space over the field of matrices F with $F = \left\{ \begin{bmatrix} r & o \\ o & r \end{bmatrix} \middle| r \in R \right\}$ under usual addition and multiplication of matrices.

25. Let v be vector space of all polynomials with real coefficients over R. Let for

$f(x) \in V$. $D^n(f) = \frac{d^n f}{du^n}$. Then

(A) D^n is a linear transformation with nullity $n - 1$

(B) D^n is a linear transformation with nullity n

(C) D^n is not a linear transformation

(D) D^n is a linear transformation with nullity $n + 1$

26. Which of the following matrices is similar to $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$:

(A) $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$

Space For Rough Work

1, 3

$\begin{vmatrix} -\lambda & 4 \\ 1 & 3-\lambda \end{vmatrix} = -\lambda(3-\lambda) - 4$

$= -3\lambda + \lambda^2 - 4$

$\Rightarrow \lambda^2 - 3\lambda - 4$

$\Rightarrow \lambda^2 - 4\lambda + \lambda - 4$

$\Rightarrow \lambda(\lambda-4) + 1(\lambda-4)$

$\begin{vmatrix} -\lambda & -3 \\ 1 & 4-\lambda \end{vmatrix}$

$-\lambda(4-\lambda) + 3 = 0$

$-4\lambda + \lambda^2 + 3 \Rightarrow \lambda^2 - 4\lambda + 3$

$\lambda^2 - 3\lambda - \lambda + 3 \Rightarrow \lambda(\lambda-3) - 1(\lambda-1)$

27. Let $V = R^3$ be a vector space over R (set of all real numbers) and T a linear operator on ∇. We call a polynomial p to be minimal polynomial of T if P(T) = 0 and there exist no polynomial q of lesser degree than p with q(T) = 0. Then the minimal polynomial of T: $(x_1, x_2, x_3) \rightarrow (x_1 + x_2, x_2 + x_3, x_3 + x_1)$.

(A) is of degree one
(B) is of degree two
(C) is of degree three
(D) does not exist

28. Let C (set of all complex numbers) be a vector space over R (set of all real number). Let $\alpha = x_1 + i\,x_2$ $\beta = y_1 + i\,y_2$ and inner product $<\alpha, \beta> = x_1 y_1 - x_2 y_1 - x_1 y_2 + 4x_2 y_2$. Then $x + iy$ and $-y + i(ix)_x$ are orthogonal.

(A) only if $y = \frac{1}{2}\left(-3 \pm \sqrt{13}\right) x$.
(B) if and only if $y = \frac{1}{2}\left(-3 \pm \sqrt{13}\right) x$.
(C) only if $y = \frac{1}{2}\left(3 \pm \sqrt{13}\right) x$.
(D) if and only if $y = \frac{1}{2}\left(3 \pm \sqrt{13}\right) x$

29. Let V be the vector space of all functions from R into C over the field C. Which of the following sets is linearly dependent in v ?

(A) $\left\{1, e^{ix}, \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} k^n\right\}$
(B) $\left\{1, \cos x, \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} x^{2n-1}\right\}$
(C) $\left\{1, \sin^2 x, 1 - \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{2n-1} \cdot x^{2n}}{(2n)!}\right\}$
(D) None of these

Space For Rough Work

$\begin{bmatrix}1&1&0\\0&1&1\\1&0&1\end{bmatrix} \Rightarrow \begin{bmatrix}1-\lambda&1&0\\0&1-\lambda&0\\1&0&1-\lambda\end{bmatrix} = (1-\lambda)\left[(1-\lambda)^2-0\right]-1\left[\right.$

$\Rightarrow (1-\lambda)^3$

$(1+\lambda)^2$ $\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix} \Rightarrow \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

$(1-\lambda)^3 \Rightarrow \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix} \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

30. Let $V = \mathbb{R}^3$ be the inner product space over $\mathbb{R}$. With respect to standard inner product. $<(x_1, x_2, x_3), (y_1, y_2, y_3)> = x_1y_1 + x_2y_2 + x_3y_3$. For what values of α, the transformation with respect to standard bases given by

$$A = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ is positive ?}$$

(A) for all $\alpha < 0$ (B) for all $\alpha \geq 1$

(C) for all $\alpha \in \mathbb{R}$ (D) None of these

31. Let F be a finite field with three elements. Then number of "unordered" bases in F^3 is

(A) $2^4 \times 3^2 \times 13$ (B) $2^5 \times 3^4 \times 13$

(C) $2^5 \times 3^2 \times 13$ (D) $2^4 \times 3^4 \times 13$

F^3

32. Which of the following matrices is diagonalizable over $\mathbb{R}$?

(A) $\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

33. Let $f(x, y) = \dfrac{x^2y}{x^2+y^2}$ for $(x, y) \neq (0, 0)$, then

(A) $\dfrac{\partial f}{\partial x}$ and f are both bounded

(B) $\dfrac{\partial f}{\partial x}$ is bounded and f is unbounded

(C) $\dfrac{\partial f}{\partial x}$ is unbounded and f is bounded

(D) $\dfrac{\partial f}{\partial x}$ and f are both unbounded

$-\lambda(2-\lambda)+3$

$(1-\lambda)^2+2=0$ $+\lambda^2$

$1+\lambda^2-2\lambda+2=0$

$\lambda^2-2\lambda+3=0$

$\dfrac{2 \pm \sqrt{4-4(3)}}{2}$

Space For Rough Work

$\begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2(1-0)-1(1-0)$

$\Rightarrow 2-1=1$

$-1(1-0)-1(1)=0$

$\Rightarrow -2$

$\begin{bmatrix} 1-\lambda & 1 \\ -3 & 2-\lambda \end{bmatrix}$ $\begin{bmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$

$(1-\lambda)(2-\lambda)+3$ $(1-\lambda)(2-\lambda)-1=0$

$2-\lambda-2\lambda+\lambda^2+3$ $2-\lambda-2\lambda+\lambda^2-1=0$

$\lambda^2-3\lambda+5$ $\lambda^2-3\lambda+1=0$

$\dfrac{3 \pm \sqrt{9-4(1)}}{2} \Rightarrow \dfrac{3 \pm \sqrt{5}}{2}$

$3 \pm \sqrt{9-4(5)}$

19

34. For $f(x, y) = \begin{cases} \dfrac{x^3 + y^3}{x^2 - y^2} & x^2 - y^2 \neq 0 \\ 0 & x^2 - y^2 = 0 \end{cases}$

the directional derivative of f at (0, 0) in the direction $\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$ is

(A) $\frac{91}{7}$ (B) $\frac{91}{5}$

(C) $\frac{91}{35}$ (D) $\frac{91}{10}$

35. For $f(x, y) = x^2 + xy + y^2 - x - 100$, the critical point of f(x, y) is

(A) $\left(\frac{2}{3}, \frac{-1}{3}\right)$ (B) $\left(\frac{-1}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{2}{3}, \frac{1}{3}\right)$ (D) $\left(\frac{-2}{3}, \frac{-1}{3}\right)$

36. The value of $\lim\limits_{(x, y) \to (2, -2)} \dfrac{\sqrt{x - y} - 2}{(x - y) - 4}$ is

$\frac{1}{2\sqrt{x-y}} \Rightarrow \frac{1}{2\sqrt{4}} \Rightarrow 1/4$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{-1}{2}$ (D) $\frac{-1}{4}$

37. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(x, y) = f(x^2 - y^2)$ then $g_{xx} + g_{yy}$ is

(A) $4(x^2 - y^2)\, f''(x^2 - y^2)$

(B) $4(x^2 + y^2)\, f''(x^2 - y^2)$

(C) $2f'(x^2 - y^2) + 4(x^2 - y^2)\, f''(x^2 - y^2)$

(D) $2(x - y)^2\, f''(x^2 - y^2)$

$\dfrac{1 - 2/\sqrt{x-y}}{\sqrt{x-y} - \frac{4}{\sqrt{x-y}}}$

$\Rightarrow 1 - \frac{2}{2}$

Space For Rough Work

$f_x = 2x + y - 1$ $f_y = x + 2y$

$2x + y - 1 = 0$ $x + 2y = 0$

$x = -2y$

$2(-2y) + y - 1 = 0$

$-4y + y - 1 = 0$

$-3y - 1 = 0$

$y = -1/3$

$y = -2(-1/3) = 2/3$

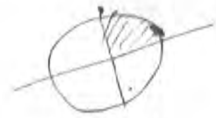
$(2/3, -1/3)$

$\Rightarrow \lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - 2}{(x+2) - 4}$

$\Rightarrow \lim \dfrac{\sqrt{2+2} - 2}{4 - 4} = 0$

38. Let $\mathbb{R}$ be a planar region bounded by the lines $x = 0$, $y = 0$ and the curve $x^2 + y^2 = 4$ in first quadrant. Let c be the boundary of R in anticlockwise direction. Then the value of

$$\oint_C x(1-y)\,dx + (x^2 - y^2)\,dy \text{ is}$$



(A) 2
(B) 4
(C) 6
(D) 8

$\Rightarrow \frac{1}{4}$ $dy = dx$

$y = x$

$x = 2$

$\Rightarrow \int_0^2 x(1-x)\,dx$

$x - x^2$

$\Rightarrow \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^2$

$\Rightarrow \frac{4}{2} - \frac{8}{3}$

$\Rightarrow \frac{12-16}{6}$

39. The last two digits in the decimal representation of 3^{128} are

(A) 9 and 1
(B) 6 and 1
(C) 5 and 1
(D) 4 and 1

40. The solution of the system of congruences $2x \equiv 1 \pmod 5$ and $3x \equiv 4 \pmod 7$ is

(A) $x \equiv 8 \pmod{35}$
(B) $x \equiv 6 \pmod{35}$
(C) $x \equiv 13 \pmod{35}$
(D) $x \equiv 20 \pmod{35}$

41. If 'a' is a primitive root of an odd prime p, then the odd power of 'a' is

(A) quadratic residue mod p if $p \equiv 1 \pmod 4$
(B) quadratic residue mod p if $p \equiv 3 \pmod 4$
(C) quadratic residue mod p for all p
(D) quadratic nonresidue mod p for all p

42. The number of elements of order 15 in the additive group $\mathbb{Z}_{30}$ is

(A) 1
(B) 15
(C) 8
(D) 10

$\mathbb{Z}_{30} \Rightarrow \frac{30}{(15, 30)}$

15×2

Space For Rough Work

$\Rightarrow \frac{30}{15}$ $\Rightarrow 15k+1 \,|\, 30$

$g = f(x^2 - y^2)$

$g_x = f'(x^2-y^2)\,2x$ $\qquad g_y = f'(x^2-y^2)(-2y)$

$\Rightarrow [x f''(x^2-y^2)\,2x + f'(x^2-y^2)]$, $g''_{yy} = -2[f''(x^2-y^2)\,y(-2y) + f'(x^2-y^2)]$

$\Rightarrow 4x^2 f''(x^2-y^2) + 2f'(x^2-y^2) + 4y^2 f''(x^2-y^2) - 2f'(x^2-y^2)$

$\Rightarrow f''(x^2-y^2)\,4(x^2+y^2)$

43. For the groups G, H and K with H being a proper subgroup of G and K being a proper subgroup of H, where o(G) = 380, o(K) = 38, the possible orders of H are

(A) 76 and 190 (B) 76 and 95

(C) 95 and 190 (D) None of these

44. The number of structurally different Abelian groups of order 1800 is

(A) 12 (B) 18

(C) 20 (D) 24

45. If G is a simple group of order 80 then G has

(A) No Sylow 2-subgroups.

(B) Exactly five Sylow 2-subgroups

(C) Exactly two Sylow 2-subgroups

(D) Exactly sixteen Sylow 2-subgroups

46. $\mathbb{Z}_5[x] / \langle x^3 + 3x + 2 \rangle$ is

(A) a field and it has 243 elements (B) Not a field but it has 243 elements

(C) a field and it has 125 elements (D) Not a field but it has 125 elements

47. In a ring $R = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$, the element $1 + 2\sqrt{-5}$ is

(A) an irreducible element

(B) a prime element

(C) both irreducible and prime element

(D) unit element

Space For Rough Work

48. If F is a field then F[x] is

(A) an Euclidean Domain (B) a Principal Ideal Domain

(C) a Unique Factorization Domain (D) All of these

49. The value of $\sum_{j=0}^{4} \cos\left(\frac{(2j+1)\pi}{11}\right)$ is

(A) 0 (B) $\frac{-3+\sqrt{11}}{2}$

(C) $\frac{3+\sqrt{11}}{2}$ (D) $\frac{1}{2}$

$\Rightarrow \cos\left(\frac{\pi}{11}\right)$

$z^4 \times \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right)$

50. The value of $\int_{|z|=1} z^4 e^{1/z}\, dz$ is

(A) 0 (B) $\frac{i\pi}{5!}$

(C) $\frac{i\pi}{12}$ (D) $\frac{i\pi}{60}$

51. Let f(z) be meromorphic in C and in $|z| < 1/3$, $f(z) = \sum_{n=1}^{\infty} n3^n z^{n-1}$.

Find $\limsup_{n \to \infty} |a_n|^{1/n}$ where $f(z) = \sum_{n=0}^{\infty} a_n(z+1)^n$.

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$

(C) $\frac{1}{3}$ (D) 1

$a_n = n3^n$

Space For Rough Work

$\cos\left(\frac{\pi}{11}\right) + \cos\left(\frac{3\pi}{11}\right) + \cos\left(\frac{5\pi}{11}\right) + \cos\left(\frac{7\pi}{11}\right) + \cos\left(\frac{9\pi}{11}\right)$

$\frac{1}{3 - 1/3}$

52. If $z = x + iy$, choose the correct option

(A) There exists a non-constant analytic function $f(z)$ in open unit disC such that $|f(z)| = e^{x^2+y^2}$.

(B) There exists a non-constant analytic function $f(z)$ in open unit disC such that $|f(z)| = \sin^2 x + \sinh^2 y + 1$.

(C) There exists a non-constant analytic function $f(z)$ in open unit disC such that $|f(z)| = x^2 + y^2 + 1$.

(D) There exists a non-constant analytic function $f(z)$ in open unit disC such that $|f(z)| = \sin 2x + \cosh^2 y + \sinh^2 y$.

53. Let $P(z)$ be a polynomial of degree n with n distant roots $z_1, z_2, \ldots, z_n$. Let

$$\frac{1}{p(z)} = \frac{A_1}{z - z_1} + \frac{A_2}{z - z_2} + \ldots + \frac{A_n}{z - z_n}$$

be the partial fraction expansion of $\frac{1}{p(z)}$ · Using the fact that $\sum_{k=1}^{n} A_k = 0$

Compute $\int_{|z|=2} \frac{1}{(z^{18} + 1)(z - 3)} dz$.

(A) 0 (B) $2\pi i$ (C) $\frac{-2\pi i}{3^{18} + 1}$ (D) $\frac{-2\pi i}{i - 3}$

54. How many roots has the equation $z^4 - 3z + 1 = 0$ in the open unit disC ?

(A) 0 (B) 1 (C) 2 (D) 3

55. Let $\gamma(t) = t + i|t|$ for $-1 \le t \le 1$. Then γ is

(A) an open and smooth path

(B) a simple and piecewise smooth path

(C) not simple but piecewise smooth path

(D) None of these

Space For Rough Work

56. Let D be simply connected domain and a function f is analytic in D. Let z_1, $z_2(\neq z_1)$ are points in D and γ is a simple path in D connecting z_1 and z_2. Then $\int_\gamma f(z)\,dz$ is

(A) 0
(B) $2\pi i$
(C) independent of path γ
(D) dependent on path γ

57. It is plainly true of any complex number z that either $\sqrt{z^2} = -z$. Identify the set of z for which the minus – sign represents the proper choice.

(A) $\{z \mid gm(z) \geq 0\}$
(B) $\{z \mid Re(z) < 0\}$
(C) $\{z \mid Re(z) < 0 \text{ and } 9\, m(z) < 0\}$
(D) $z \mid Re(z) > 0 \text{ and } gm(z) < 0\}$

58. Choose the wrong option.

(A) An open unit disC is a simply connected domain.
(B) The whole complex plane is itself a simply connected domain.
(C) Frontier of a closed unit disC is a simply connected domain.
(D) The set $S = \{z \mid |z| < 1\} \cup \{z \mid |z-2| < \frac{3}{2}\}$ is a simply connected domain.

59. The lower limit topology on R is generated by

(A) $\{(a, b) : a < b\}$
(B) $\{[a, b) : a < b\}$
(C) $\{(a, b] : a < b\}$
(D) $\{[a, b] : a < b\}$

60. If π_1 is the projection of $X \times Y$ onto X and if U is an open set of X, then $\pi_1^{-1}(U)$ is

(A) U
(B) $X \times U$
(C) $U \times Y$
(D) X

Space For Rough Work

61. The closure of $\left(0, \frac{1}{2}\right)$ in (0, 1] is

(A) $\left[0, \frac{1}{2}\right]$

(B) $\left(0, \frac{1}{2}\right]$

(C) $\left(0, \frac{1}{2}\right)$

(D) $\left[0, \frac{1}{2}\right)$

62. With respect to the standard metric on $\mathbb{R}'$, the ball B (2.8, 0.3) is

(A) [2.5, 3.1]

(B) (2.5, 3.1)

(C) (2.8, 3.1)

(D) [2.8, 3.1]

63. If y is compact, then the projection map $\pi_1 : X \times Y \rightarrow X$ is

(A) a closed map

(B) an open map

(C) a continuous map

(D) uniform continuous map

64. Which of the following is an equivalent condition for X to be connected ?

(A) X has a non empty set which is both closed and open.

(B) $X = A \cup B$ where A and B are two non empty disjoint sets.

(C) $X = A \cup B$ where A is a non empty closed set and B is a non empty open set and $A \cap B = \phi$.

(D) The only subsets of X that are both open and closed in X are empty set and X itself.

65. Consider $\left|\frac{dy}{dx}\right| + |y| + a = 0$, $a \in R$, then the real valued solution passing through point $(b, c) \in R^2$ where $a > 0$ has

(A) unique real solution $\forall\ (b, 0) \in R^2$

(B) no real solution $\forall\ (b, c) \in R^2,\ c \neq 0$

(C) no real solution $\forall\ (b, c) \in R^2$

(D) unique real solution $y(x) = -a,\ \forall\ (b, -a) \in R^2$

Space For Rough Work

66. If $y(x) = x|x|$ is solution of n^{th} order differential equation defined $\forall\ x \in R$, then possible value(s) of n is/are

(A) $n \geq 2$ (B) $n = 1$

(C) $n = 3$ (D) $n = 3/2$

67. The difference of any two solutions of $y'' + p(x)y' + q(x)y = f(x)$ is a solution of its reduced equation. This statement is

(A) Always true (B) Never true

(C) Sometimes true (D) Partially false

68. The Wronskion of the functions $y_1 = x^2$ and $y_2 = x|x|$ at x is

(A) $w(y_1, y_2)(x) = 1$ (B) $w(y_1, y_2)(x) = 0$

(C) $w(y_1, y_2)(x) = -1$ (D) $w(y_1, y_2)(x) = -x^2|x|$

69. The eigen values of any regular Sturm Liouville boundary value problem are

(A) always real (B) always complex

(C) either real or complex (D) All of these

70. Eigen values and eigen functions of a boundary value problem $x^{11} + \lambda x = 0$, $x(0) = 0$, $x'(\pi) = 0$ are

(A) $\lambda = -\left(\frac{2n+1}{2}\right)^2$ and $x_n(t) = \cos\left(\frac{2n+1}{2}t\right)$

(B) $\lambda = -\left(\frac{2n+1}{2}\right)^2$ and $x_n(t) = \sin\left(\frac{2n+1}{2}t\right)$

(C) $\lambda = \left(\frac{2n+1}{2}\right)^2$ and $x_n(t) = \cos\left(\frac{2n+1}{2}t\right)$

(D) $\lambda = \left(\frac{2n+1}{2}\right)^2$ and $x_n(t) = \sin\left(\frac{2n+1}{2}t\right)$

Space For Rough Work

71. Green's function of $x'' + x = f(t)$ satisfying boundary conditions $x(0) = 0, x\left(\frac{\pi}{2}\right) = 0$ is

(A) $G(t, s) = \begin{cases} \cos t \sin s & \text{if } t < s \\ -\sin t \cos s & \text{if } t > s \end{cases}$

(B) $G(t, s) = \begin{cases} \sin s \cos t & \text{if } t < s \\ \cos s \sin t & \text{if } t > s \end{cases}$

(C) $G(t, s) = \begin{cases} \sin t \cos s & \text{if } t < s \\ \sin s \cos t & \text{if } t > s \end{cases}$

(D) None of these

72. The solution of the differential equation $(x \sin (y/x))\, dy - (y \sin (y/x) - x)\, dx = 0$ is

(A) $\cos (y/x) - \frac{1}{x} = 0$

(B) $\sin (y/x) + \frac{1}{x} = 0$

(C) $\cos (y/x) - \log x = \text{constant}$

(D) $\sin (y/x) - \log x = \text{constant}$

73. The n^{th} order homogeneous linear differential equation $y^{(n)} + p_0 y^{(n-1)} + \ldots + p_n y = 0$ has general solution if the coefficients $p_0(x), p_1(x), \ldots, p_n(x)$ on some interval I are

(A) continuous

(B) discontinuous

(C) need not to be continuous

(D) None of these

74. The solution of the nonhomogeneous equation $y'' + a^2 y = \sec ax$ is

(A) $y = \frac{1}{a^2}(\cos ax \log (\cos ax) + C_4 \cos ax + C_3 \sin ax + \frac{x}{a} \sin ax$

(B) $y = \cos ax \log (\cos ax) + C_4 \cos ax + C_3 \sin ax + \frac{x}{a} \sin ax$

(C) $y = \frac{1}{a^2}(\sin ax \log (\cos ax) + C_4 \sin ax + C_3 \cos ax + \frac{x}{a} \sin ax)$

(D) $y = \cos ax \log (\cos ax) + C4 \sin ax + C_3 \sin ax + \frac{x}{a} \cos ax$

$x \sin(y/x)\, dy = (y \sin(y/x) - x)$

$\frac{dy}{dx} = \frac{y \sin(y/x) - x}{x \sin(y/x)}$

$\frac{y/x \sin y/x - }{\sin(y/x)}$

$\frac{v \sin v - }{\sin v}$

Space For Rough Work

$y = c_1 \cos x + c_2 \sin x, \quad x(0) = 0$

$f(t)=0 \quad u = \sin x$

$v = \cos x$

$\frac{1}{D^2 + a^2} \sec ax$

$v = y/x$

$v + x\frac{dv}{dx} = \frac{v \sin v}{\sin v}$

$y = vx$

$dy = v + x\frac{dv}{dx}$

$x\frac{dv}{dx} = -$

$\cos(y/x) - \log$

75. Which of the following statements are true ?

(i) singular solution contains no arbitrary constant.

(ii) singular solution can be obtained from complete primitive.

(A) both (i) & (ii) are true (B) both (i) & (ii) are false

(C) (i) is true but (ii) is false (D) (i) is false, but (ii) is true

76. The general integral of a partial differential equation $\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 5z + \tan(y - 3x)$ is

(A) $y - 3x = f(e^{-5x}(5z + \tan(y - 3x)))$ (B) $y - 3x = f(e^{-5x}(5z + \tan x))$

(C) $y - 3x = f(e^{-5x}(5z - \tan 3x))$ (D) None of these

$p+3q = 5z+\tan(y-3x)$

$a+3b = 5z+\tan(b-3a)$

77. The complete integral of the equation

$$2xz - \left(\frac{\partial z}{dx}\right)x^2 - 2\left(\frac{\partial z}{dy}\right)xy + \left(\frac{\partial z}{dx}\right)\left(\frac{\partial z}{dy}\right) = 0 \text{ is}$$

(A) $z = ay + b(x^2 + a)$ (B) $z = ay + b(x^2 - a)$

(C) $z = ax + b(x^2 - a^2)$ (D) $z = ax + b(x^2 + a^2)$

78. Which of the following function satisfies the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) + z =$$

(A) $z = e^{-x} f_1(y - x) + xe^{-x} f_2(y - x)$ (B) $z = 2e^{-x} f_1(y + x)$

(C) $z = f_1(y - x) + x f_2(y - x)$ (D) None of these

79. For a given wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ which of the following are true ?

(i) The general solution is $u(x, t) = \phi(x + ct) + \phi(x - ct)$

(ii) No general solution exist.

(iii) D' Alembert's solution of the given equation is $u(x, t) = f(x + ct) + f(x - ct)$

(iv) No D' Alembert's solution exist.

(A) Both (i) & (ii) (B) Both (i) & (iii)

(C) Both (ii) & (iv) (D) All are true

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80. The solution of the equation $\frac{y^2z}{x}p + xzq = y^2$ is

(A) $\phi(x^3 - y^3, x^2 - z^2)$
(B) $\phi(x^2 - y^2, x^2 - z^2)$
(C) $\phi(x^3 + y^3, x^2 + z^2)$
(D) $\phi(x^2 + y^2, x^2 + z^2)$

$\frac{dx}{\frac{y^2z}{x}} = \frac{dy}{xz} = \frac{dz}{y^2}$

$x\frac{dx}{y^2z} = \frac{dz}{y^2}$

$x\,dx = z\,dz$

$x^2 - z^2 =$

81. A surface passing through the two lines $z = x = 0$, $z - 1 = x - y = 0$ and satisfying $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$ is

(A) $z = \frac{3y}{y + 2x}$
(B) $z = \frac{3x}{y + 2x}$
(C) $z = \frac{2}{y + 3x}$
(D) $z = \frac{3}{x + 3y}$

$x\frac{dx}{y^2z} = \frac{dy}{xz}$

$x^2\,dx = y^2$

$x^3 - y^3$

82. For the equation

$$\sin^2 x\, u_{xx} + \sin 2x\, u_{xy} + \cos^2 x\, u_{yy} = x$$

which of the following are true ?

(i) parabolic equation
(ii) hyperbolic equation
(iii) canonical form is $\cos^2 x\, u_{\eta\eta} + u_{\epsilon} = u_{\epsilon\eta}$
(iv) canonical form is $u_{\eta\eta} = \frac{\sin^{-1}(e^{\eta-\epsilon}) - u_{\epsilon}}{1 - e^{2(\eta-\epsilon)}}$

(A) both (i) and (iii)
(B) both (ii) and (iii)
(C) both (i) and (iv)
(D) both (ii) and (iv)

$m^2 - 4m + 4$

$m^2 - 2m - 2m + 4$

$m(m-2) - 2(m-2)$

$m = 2, 2$

83. Given that f(1) = 1, f(2) = 3, f(3) = 8, f(4) = 15 and f(5) = 25, then Δ^4 f(1) is

(A) 1
(B) 2
(C) −1
(D) 0

84. If Δ and ∇ denote the forward and backward difference operators, then $\Delta + \nabla$ is equal to

(A) $(E - 1)\nabla$
(B) $(E - 1)\Delta$
(C) $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$
(D) $\frac{\Delta}{\nabla} + \frac{\nabla}{\Delta}$

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$\Delta = E - 1$

$E = 1 + \Delta$

$E = (1 + \nabla)^{-1}$

$E(1 + \nabla) = 1$

$1 + \nabla = E^{-1}$

$\boxed{\nabla = E^{-1} - 1}$

$\Delta = E - 1 + E^{-1} - 1$

$= E + E^{-1} - 2$

$\Delta = 1 - E$

$= 1 - E$

$\Delta = E - 1$

$E(1 + \nabla)$

$E + E($

85. If the interval (2, 6) is subdivided into four subintervals of equal length, the approximate value of

$$\int_{2}^{6} \frac{dx}{x^2 - x} \text{ using Simpson's rule is}$$

(A) 0.3222 (B) 0.2333

(C) 0.5222 (D) 0.2555

86. Newton-Raphson method is used to calculate $\sqrt[3]{65}$ by solving $x^3 = 65$. If $x_0 = 4$ is taken as the initial approximation, the first approximation x_1 is

(A) $\frac{65}{16}$ (B) $\frac{131}{32}$

(C) $\frac{191}{48}$ (D) $\frac{193}{48}$

$x_1 = 4 - \frac{x^3 - 65}{3x^2}$

$= 4 - \frac{64 - 65}{3(16)}$

$= 4 + \frac{1}{48}$

$\Rightarrow \frac{193}{48}$

87. The extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3} dx$ are

(A) $y = ax^4 + b$, a, b constants (B) $y = ax^3 + b$, a, b constants

(C) $y = ax^2 + bx + c$, a, b, c constants (D) $y = ax + b$, a, b constants

88. The extremal of the functional $\int_{0}^{1} (1 + y''^2)\, dx$ subject to the conditions

$y(0) = 0, y'(0) = 1$

$y(1) = 1, y'(1) = 1$

is

(A) $y = x^2$ (B) $y = x$

(C) $y = x^2 + x - 2$ (D) $y = x^2 - x + 1$

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$y = ax + b$

$y(0) = \boxed{0 = b} \Rightarrow \boxed{y = x}$

$y' = a = 1$

$y(1) = 1 =$

$\frac{\partial f}{\partial y'} = K$

$\frac{2y'}{x^3} = K \Rightarrow 2y' = Kx^3$

$\therefore y' = \frac{K}{2} x^3$

$y = \frac{K}{8} x^4$

89. The extremal of the functional $\int_0^1 (y'^2 - y^2)\, dx$ subject to $y(0) = 0,\ y(1) = 1$ is

(A) $\sin\left(\frac{\pi}{2} x\right)$

(B) x

(C) $\frac{\sin x}{\sin 1}$

(D) $2x^2 - x$

90. Define $f(t) = \begin{cases} A, & |t| < T \\ 0 & \text{otherwise} \end{cases}$

If F(w) denotes the Fourier transform of f(t), then

(A) $|F(\omega)|^2 = \frac{A^2 \sin^2(\omega T)}{\omega^2}$

(B) $|F(\omega)| = \frac{A \sin(\omega T)}{\omega^2}$

(C) $|F(\omega)|^2 = \frac{4A^2 \sin^2(\omega T)}{\omega^2 T^2}$

(D) $|F(\omega)|^2 = \frac{4A^2 \sin^2(\omega T)}{\omega^2}$

91. Let f(f) denote the Fourier transform of f and a, b reals with $a \neq 0$.

If $g(x) = f(ax + b)$, then

(A) $f(g(w)) = \frac{1}{|a|} e^{iwb/a} f(f)\left(\frac{w}{a}\right)$

(B) $f(g(w)) = \frac{1}{|a|} e^{iwb/a} f(f)\,(aw)$

(C) $f(g(w)) = \frac{1}{|a|} e^{iwa/b} f(f)\left(\frac{w}{a}\right)$

(D) $f(g(w)) = \frac{1}{|b|} e^{iwb/a} f(f)\left(\frac{w}{a}\right)$

92. Let F(w) denote the Fourier transform of f(t). Then

(A) $\lim_{w \to +\infty} F(w) = c,\ c \neq 0$ but $\lim_{w \to -\infty} F(w) = 0$

(B) $\lim_{w \to +\infty} F(w) = 0$, but $\lim_{w \to -\infty} F(w) = c,\ c \neq 0$

(C) $\lim_{w \to \pm\infty} F(w) = 0$

(D) None of these

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$\frac{1}{2}\sin^2(\frac{1}{4}) = 1 +$

93. The Laplace transform of $\int_0^t \frac{\sin u}{u}\,du$ is

(A) $\frac{1}{s}\tan^{-1}\left(\frac{1}{s}\right)$

(B) $\frac{1}{s}\tan^{-1}(s)$

(C) $s\tan^{-1}\left(\frac{1}{s}\right)$

(D) $s\tan^{-1}(s)$

94. The inverse Laplace transform of $\frac{s}{(s^2+1)^2}$ is

(A) $2t \sin t$

(B) $t \sin t$

(C) $\frac{1}{2} t \sin t$

(D) $t \sin(2t)$

95. If $f(t) = t^2$, $g(t) = \sin t$ and $h(t) = (f * g)(t)$, the convolution of f(t) and g(t), then h(t) =

(A) $t^2 + 2\cos t + 2$

(B) $t^2 - 2\cos t - 2$

(C) $t^2 - 2\cos t + 2$

(D) $t^2 + 2\cos t - 2$

96. A rope of length $2l$ meter suspended between two points at the same level, and the lowest point of the rope is "b" meter below the point of suspension. The weight of the rope per meter of its length is "w". the horizontal component of the tension is

(A) $\frac{wl^2}{2b}$

(B) $\frac{w(l^2 + b^2)}{2b}$

(C) $\frac{wb^2}{2l}$

(D) $\frac{w(l^2 - b^2)}{2b}$

97. Two men carry a load of 224 kg wt, which hangs from a light pole of length 8 m. Each end of pole rests on the shoulder of one of the men. The point from which the load is hung is 2 m nearer to one man than the other. The load on each shoulder

(A) 140 kg wt, 84 kg wt.

(B) 144 kg wt, 80 kg wt.

(C) 164 kg wt, 60 kg wt.

(D) 174 kg wt, 50 kg wt.

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98. A particle is under simple harmonic motion in a straight line. If "v" be its velocity when at a distance "x" from a fixed point in the line and $\upsilon^2 = \alpha - \beta x^2$, where α, β are constants. Then period and amplitude of oscillation respectively –

(A) $\frac{2\pi}{\sqrt{\beta}}, \sqrt{\frac{\beta}{\alpha}}$

(B) $\frac{2\pi}{\sqrt{\alpha}}, \sqrt{\frac{\alpha}{\beta}}$

(C) $\frac{2\pi}{\sqrt{\beta}}, \sqrt{\frac{\alpha}{\beta}}$

(D) $\frac{2\pi}{\sqrt{\alpha}}, \sqrt{\frac{\beta}{\alpha}}$

99. In which of the following conditions, the total linear momentum of the system remains constant ?

(A) If the resultant external force acting on the system of particle is zero.

(B) If the resultant external force acting on the system of particles is positive.

(C) If the resultant external force acting on the system of particles is negative.

(D) None of these.

100. A particle is placed in a region with the potential $V(x) = 1/2\ kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$. Then

(A) $x = 0$ and $x = \frac{k}{\lambda}$ are the points of stable equilibrium.

(B) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium.

(C) $x = 0$ and $x = \frac{k}{\lambda}$ are the points of unstable equilibrium.

(D) There are no points of stable or unstable equilibrium.

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101. A planet of mass "m" moves in a circular orbit of radius r_0 in the gravitational potential $V_{(r)} = -\frac{k}{r}$, where k is a positive constant. The orbit angular momentum of the planet is

(A) $2 r_0 km$

(B) $\sqrt{2 r_0 km}$

(C) $r_0 km$

(D) $\sqrt{r_0 km}$

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102. The law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be described as

(A) $p \alpha \frac{1}{r^{2n+3}}$

(B) $p \alpha \frac{1}{r^{2n-3}}$

(C) $p \alpha \frac{1}{r^{2n}}$

(D) $p \alpha \frac{1}{r^{n}}$

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103. For some distribution, mean is 12 and Median is 21, then mode is

(A) 16.5

(B) 39

(C) Insufficient Data

(D) None of these

104. Any four Natural Numbers are selected and multiplied, the probability that last digit will be 1, 3, 5 or 7 is

(A) $\frac{16}{625}$

(B) $\frac{1}{16}$

(C) $\frac{81}{1000}$

(D) $\frac{27}{125}$

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105. A number is selected from first 100 natural numbers, then probability that x satisfies $x + \frac{100}{x} > 50$

(A) $\frac{17}{100}$

(B) $\frac{9}{20}$

(C) $\frac{83}{100}$

(D) $\frac{11}{20}$

106. The probability distribution function is given by $p(x) = \frac{k}{2^x}, x = 1, 2, \ldots$

Then (i) value of k

(ii) expedition of odd natural numbers

(A) (i) $\frac{1}{2}$ (ii) $\frac{9}{10}$

(B) (i) $\frac{3}{4}$ (ii) 3

(C) (i) $\frac{1}{4}$ (ii) $\frac{9}{10}$

(D) (i) $\frac{1}{2}$ (ii) 3

107. For bivariates X and Y, with X : 0, 1, 2 Y : 0, 1, 2, having Marginal Distribution of X is $\left\{\frac{1}{2}, \frac{3}{10}, \frac{1}{5}\right\}$ and Marginal Distribution of Y is $\left\{\frac{3}{10}, \frac{3}{10}, \frac{2}{5}\right\}$. If expertation of XY [E(xy)] is $\frac{77}{100}$, then Random variables x & y are

(A) Dependent

(B) Independent

(C) Insufficient Data

(D) None of these

108. If a fair die is rolled more than 36 times, the probability of Number of 6 lies between $\frac{1}{6}\sqrt{n} - n$ and $\frac{1}{6}\sqrt{n} + n$ is atleast

(A) $\frac{1}{36}$

(B) $\frac{35}{36}$

(C) $\frac{31}{36}$

(D) $\frac{5}{36}$

109. Which of the following Reasons causing degeneracy in Transportation problem where m represents number of Rows and n represents number of columns ?

(A) When number of allocation is m + n – 1.

(B) Two or more occupied cells will become unoccupied simultaneously.

(C) Number of allocation is less than m + n – 1.

(D) Loop cannot be drawn without using unoccupied cell, except starting cell of loop.

110. For linear programming,

Maximize $z = 15x_1 + 10x_2$,

subject to conditions $4x_1 + 6x_2 \le 360$

$3x_1 \le 180$

$5x_2 \le 200, x_1, x_2 \ge 0$

then (i) value of Maximum z

(ii) values of Basic variables x_1 and x_2, are

$\frac{4x_1}{360} + \frac{6}{360}x_2 \le 1$

$\frac{x_1}{90} + \frac{x_2}{60} \le 1$

(90, 0) (0, 60)

$\frac{3x}{180} \le 1$

(60, 0)

(0, 40)

(A) (i) $z = 1500$ (ii) $x_1 = 100, x_2 = 0$

(B) (i) $z = 1250$ (ii) $x_1 = 0, x_2 = 125$

(C) (i) $z = 1100$ (ii) $x_1 = 60, x_2 = 20$

(D) (i) $z = 1000$ (ii) $x_1 = 40, x_2 = 40$

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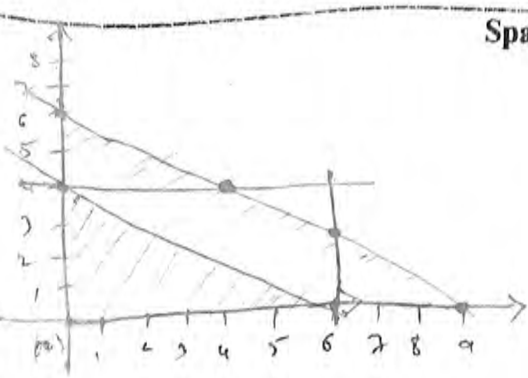

(0, 0), (0, 40), (60, 0)

60 × 15

Z = 15(0) + 10(40) = 400

Z = 15 × 60

$\Rightarrow 15x_1$

111. For primal problem

Minimize, $z = 3x_2 + 5x_3$

Subject to conditions $\quad x_1 + x_2 \geq 2$

$2x_1 + x_2 + 6x_3 \leq 6$

with $\quad x_1, x_2, x_3 \geq 0$

$\frac{x_1}{2} + \frac{x_2}{2} \geq 1$

$z = 3x_2 + 5x_3$

$(0, 2)$

$(2, 0)$

its duality and subject to conditions are

(i) $\quad \text{Min}(z_w) = -2\,w_1 + 6w_2 + 4w_3 - 4w_4$

(ii) $\quad$ Subject to conditions :

$-w_1 + 2w_2 + w_3 - w_4 \geq 0$

$-w_1 + w_2 - w_3 + w_4 \geq -3$

$6w_2 + 3\,w_3 - 3\,w_4 \geq -5$

with $w_1, w_2, w_3, w_4 \geq 0$

then

(A) both (i) and (ii) are correct.

(B) both (i) and (ii) are incorrect

(C) only one of (i) and (ii) are correct

(D) None of these

112. If $\{x_{13}, x_{22}, x_{23} = 10, x_{31}, x_{32}, x_{34}\}$ is the set of basic variables of balanced transportation from origin to destination with cost matrix.

	D_1	D_2	D_3	D_4	Available
01	6	2	–1	0	10
02	4	2	2	3	$\lambda + 5$
03	3	1	2	1	3λ
Demand	10	$M + 5$	$M - 5$	15	

With $\lambda, M \in \mathbb{R}$, the value of x_{32}.

(A) 5

(B) 10

(C) 15

(D) 30

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113. With regarding to Game Matrix

		B_1	B_2	B_3
	A_1	2	4	2
A	A_2	1	–5	–4
	A_3	2	6	–2

then (i) Value of the game

(ii) Strategy for A & B

(A) (i) value of Game = 2, (ii) Strategy for A and B is $(A_1\ B_1)$

(B) (i) value of Game = 3, (ii) Strategy for A and B is $(A_3\ B_1)$

(C) (i) value of Game = 2, (ii) Strategy for A and B is $(A_1\ B_1)$ or $(A_1\ B_3)$ or $(A_3\ B_1)$

(D) (i) value of Game = 3, (ii) Strategy for A and B is $(A_1\ B_3)$

114. Which of the following statements is not true ?

(A) Every totally ordered set is a distributive lattice.

(B) A lattice with less than five elements is distributive.

(C) Every distributive lattice is a bounded lattice.

(D) Every sublattice of a distributive lattice is distributive.

115. Let S = {p, q, r, s, t} and

R = {(p,p), (p, q), (p, r), (p, s), (p, t), (q, q),
(q, s), (q, t), (s, s), (s, t), (r, r),
(r, t), (t, t)} $\subseteq$ S × S.

Then which of the following is true ?

(A) (S, R) is a complemented lattice.
(B) (S, R) is a distributive lattice.
(C) (S, R) is a Boolean Algebra.
(D) (S, R) is a modular lattice.

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116. Which of the following sets must be included into

$S = \{\{1, 2\}, \{1, 2, 3\}, \{1, 3, 5\}, \{1, 2, 4\}, \{1, 2, 3, 4, 5\}\}$

to make S a complete lattice under the partial order defined by "contained in" for sets ?

(A) $\{1\}, \{2, 3\}$

(B) $\{1\}$

(C) $\{1\}, \{1,3\}$

(D) $\{1\}, \{1,3\}, \{1,2, 3, 4\}, \{1,2, 3, 5\}$

117. Consider a set $S = \{x, b_1, b_2 \ldots, b_n, y\}$ and a partial order $\leq$ defined on S as $x \leq b_i$ for each i and $b_i \leq y$ for each i, where $n \geq 1$.

The number of total orders on S which contain the partial order $\leq$ is

(A) n

(B) n^2

(C) n!

(D) n^3

118. In the partially ordered set $(P(\{a, b, c\}), \subseteq)$, ϕ is

(A) the least element but not minimal.

(B) the minimal element but not the least.

(C) neither least element nor minimal element.

(D) both least element and minimal element.

119. Which of the following is not true ?

(A) Every finite lattice is bounded.

(B) Every distributive lattice is modular.

(C) Every modular lattice is distributive.

(D) Every bounded lattice in which every element has a complement is a complemented lattice.

120. The number of ways in which Raju, Ravi, Rishi, Roopa, Ramu and Raghu can be seated in a round table such that Raju and Ravi must always sit together is

(A) 12

(B) 24

(C) 36

(D) 48

121. Let G be a graph having 40 edges such that there are seven vertices of degree 2, two vertices of degree 5 and the remaining vertices are of degree 7. Then the order of G is

(A) 13 (B) 15

(C) 17 (D) 19

= 17 $\frac{7-14}{2}$

122. A complete graph k_p is planar if and only if

(A) $p < 5$ (B) $p = 5$

(C) $p > 5$ (D) None of these

123. The total number of edges in a forest G with 60 vertices and 20 components is

(A) 60 (B) 40

(C) 59 (D) 61

⇒ 60 - 20 = 20

124. The graph $K_{m, n}$, $m \geq 2$, $n \geq 2$ is Eulerian if

(A) Both m and n are odd integers. (B) One of m and n is odd integer.

(C) None of m and n is odd integer. (D) None of these

125. The sum of the entries in a row of the adjacency matrix of a graph G is the

(A) eccentricity of the corresponding vertex.

(B) degree of the corresponding vertex.

(C) weight of the corresponding vertex.

(D) None of these

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