68th BPSC Mains

Previous Year Paper

(Maths Optional)

18 May, 2023

1. If u, v and w are the components of velocity along x-axis, y-axis and z-axis, the equation of continuity in the form of incompressible fluid is

(A) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

(B) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{\partial^2 w}{\partial z^2}$

(C) $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}$

(D) None of the above

2. A sink is

(A) a source of negative strength

(B) a source of positive strength

(C) a point at which the fluid continuously annihilates

(D) Both (A) and (C)

3. Strength of a source is

(A) the total volume of flow per unit time

(B) the total velocity of flow across the surrounding

(C) the total force across the source

(D) None of the above

4. If the speed is everywhere same, the streamlines are

(A) circular

(B) elliptic

(C) parabolic

(D) straight lines

5. The moment of inertia of a right circular cylinder about its axis is (where M is mass of the cylinder and a is radius of the base)

(A) Ma^2

(B) $\frac{3}{2}Ma^2$

(C) $\frac{Ma^2}{2}$

(D) $2Ma^2$

6. The whole pressure of a heavy homogeneous liquid on a plane area is equal to

(A) the product of the area and the pressure, at its centre of gravity

(B) the sum of the area and the pressure, at its centre of gravity

(C) the pressure, at its centre of gravity

(D) None of the above

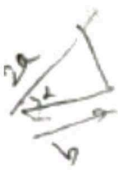
7. A uniform beam of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg distant b from the wall. The virtual work is in position of equilibrium. The beam is inclined to the wall at an angle

(A) $\cos^{-1}\left(\frac{b}{a}\right)^{1/3}$

(B) $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$

(C) $\tan^{-1}\left(\frac{b}{a}\right)^{1/3}$

(D) $\sec^{-1}\left(\frac{b}{a}\right)^{1/3}$



8. If T is the tension at any point P of a common catenary, and T_0 that at the lowest point A, then

(A) $T^2 + T_0^2 = W^2$

(B) $T + T_0 = W$

(C) $T - T_0 = W$

(D) $T^2 - T_0^2 = W^2$

where W is the weight of the arc AP of the catenary.

9. The locus of points, such that every force system is equivalent to a single force and a couple whose moment is parallel to the single force, is

(A) central axis

(B) central point

(C) central force

(D) None of the above

10. The null line is a line

(A) about which the total force is zero

(B) about which the moment of force system is zero

(C) about which the moment of force system is non-zero

(D) None of the above

11. A particle is moving in simple harmonic motion and while making an excursion from one position of rest to the other, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3. The time of complete oscillation is

(A) $\dfrac{\pi}{\cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)}$

(B) $\dfrac{\pi}{\sin^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)}$

(C) $\dfrac{2\pi}{\cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)}$

(D) $\dfrac{2\pi}{\sin^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)}$

12. The set of all non-singular square matrices of order n with real elements is

(A) an Abelian group with respect to multiplication of matrices

(B) a group with respect to multiplication of matrices

(C) not a semigroup with respect to multiplication of matrices

(D) None of the above

13. The set of fourth roots of unity

(A) does not form a group with respect to addition

(B) forms an Abelian group with respect to addition

(C) forms an Abelian group with respect to multiplication

(D) forms a group with respect to addition

14. Let G be a group of order $2p$, where p is prime, then G has a normal subgroup of order

(A) p

(B) $2p$

(C) 0

(D) $3p$

15. Let $G = \{1, -1, i, -i\}$ be the multiplicative group and $H = \{1, -1\}$ be the subgroup of G, then

(A) H is not normal subgroup of G

(B) H is not subset of G

(C) H is the normal subgroup of G

(D) None of the above

16. Every quotient group of a cyclic group is

(A) cyclic

(B) not cyclic

(C) sometimes cyclic and sometimes not cyclic

(D) None of the above

17. If $f : G \to G'$ is a homomorphism of groups and e, e' are the identities of G and G' respectively, then

(A) $f(e) = e$

(B) $f(e) = e'$

(C) $f(e) = 1$

(D) $f(e') = e$

1

18. $|\underline{n}$ permutations on n symbols have

(A) $|\underline{n}$ even permutations

(B) $|\underline{n}$ odd permutations

(C) $\frac{|\underline{n}}{2}$ even permutations and $\frac{|\underline{n}}{2}$ odd permutations

(D) $|\underline{n}$ even permutations and $|\underline{n}$ odd permutations

19. Which of the following structures is ***not*** a ring?

(A) $(2I, +, \cdot)$

(B) $(N, +, \cdot)$

(C) $(R, +, \cdot)$

(D) $(C, +, \cdot)$

20. If the two spheres

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$
and
$$x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0$$

cut orthogonally, then

(A) $uu' + vv' + ww' = d + d'$

(B) $uu' + vv' + ww' = dd'$

(C) $2uu' + 2vv' + 2ww' = d + d'$

(D) $2uu' + 2vv' + 2ww' = dd'$

21. Applying mean value theorem to the function $f(x) = (x-2)(x-3)$ in the interval $[1, 2]$, the value of c is

(A) 2

(B) 3

(C) 1

(D) $\frac{3}{2}$

22. If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

is equal to

(A) 2

(B) 1

(C) 3

(D) 0

23. If

$$u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$$

then the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is equal to

(A) 0

(B) 3

(C) 1

(D) 2

24. The equation of the plane which cuts the paraboloid $x^2 - 2y^2 = 3z$ in the conic with centre (1, 2, 3) is

(A) $x - 4y - 2z + 20 = 0$

(B) $2x - 8y - 3z + 23 = 0$

(C) $x + 8y - z + 20 = 0$

(D) $2x + 8y - 3z - 23 = 0$

25. From the regression equations $8x - 10y = -66$ and $40x - 18y = 214$, the mean values of x and y series are respectively

(A) 15 and 15

(B) 18 and 12

(C) 13 and 17

(D) 10 and 20

26. One ticket is drawn at random from a bag containing 30 tickets numbered from 1 to 30. The probability that it is a multiple of 3 or 5 is

(A) $\frac{2}{15}$

(B) $\frac{4}{15}$

(C) $\frac{11}{15}$

(D) $\frac{7}{15}$

27. A bag contains 6 red and 3 white balls. 4 balls are drawn out one by one and not replaced. The probability that they are alternatively of different colours is

(A) $\frac{5}{42}$

(B) $\frac{5}{84}$

(C) $\frac{5}{21}$

(D) $\frac{5}{63}$

3/28

28. A class consists of 80 students, 25 of them are girls and 55 boys, 10 of them are rich and remaining poor, 20 of them are fair complexioned. The probability of selecting a fair-complexioned rich girl is

(A) $\frac{5}{512}$ 3/8 6/7 2/6 5/5

(B) $\frac{5}{256}$

(C) $\frac{5}{128}$

(D) $\frac{5}{64}$

29. How many tosses of a coin are needed so that the probability of getting at least one head is 0·875?

(A) 10

(B) 5

(C) 2

(D) 3

30. If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$, then the standard deviation of the distribution is

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) $\sqrt{5}$

(D) $\sqrt{7}$

31. The normal equations of the curve $y = a + bx^2$ are

(A) $\Sigma y = a\Sigma x + b\Sigma x^2$ and $\Sigma xy = a\Sigma x + b\Sigma x^3$

(B) $\Sigma y = na + b\Sigma x^2$ and $\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4$

(C) $\Sigma y = a\Sigma x^2 + b\Sigma x^3$ and $\Sigma xy = a\Sigma x + b\Sigma x^4$

(D) $\Sigma x = n\Sigma y + b\Sigma xy$ and $\Sigma x^2 y = a\Sigma y^2 + b\Sigma xy$

32. The coefficient of correlation is

(A) the algebraic mean of regression coefficients

(B) the geometrical mean of regression coefficients

(C) the product of regression coefficients

(D) the sum of regression coefficients

33. The coefficient of range is

(A) always positive

(B) always negative

(C) always less than one

(D) Both (A) and (C)

34. In a moderately skewed distribution, if mean = 150, mode = 140 and standard deviation = 45, then the coefficient of skewness is

(A) 0·22

(B) 0·44

(C) 0·66

(D) 0·77

35. The first four central moments of a distribution are 0, 2·5, 0·7 and 12·50. The kurtosis is

(A) platykurtic

(B) mesokurtic

(C) leptokurtic

(D) None of the above

36. The value of the game, whose pay-off matrix is

$$\begin{array}{cc|ccc|} & & & B & \\ & & I & II & III \\ \hline & I & -3 & -2 & 6 \\ A & II & 2 & 0 & 2 \\ & III & 5 & -2 & -4 \\ \hline \end{array}$$

is

(A) -4

(B) 0

(C) -3

(D) 6

37. The solution to a transportation problem with m rows (supplies) and n columns (destinations) is feasible if the number of positive allocations is

(A) $m+n+1$

(B) $m+n$

(C) $m \times n$

(D) $m+n-1$

38. A game is said to be fair if

(A) both upper and lower values of the game are same and zero

(B) upper and lower values of the game are not equal

(C) upper value is more than lower value of the game

(D) None of the above

39. The size of the pay-off matrix of a game can be reduced by using the principle of

(A) game inversion

(B) rotation reduction

(C) dominance

(D) game transpose

40. The entering variable in the sensitivity analysis of objective function coefficients is always a

(A) decision variable

(B) non-basic variable

(C) basic variable

(D) slack variable

41. If there were n workers and n jobs, there would be

(A) $n!$ solutions

(B) $(n-1)!$ solutions

(C) $(n!)^n$ solutions

(D) n solutions

42. For a salesman who has to visit n cities, the number of ways of his tour plan is

(A) $n!$

(B) $(n+1)!$

(C) $(n-1)!$

(D) n

43. For a maximization problem, the objective function coefficient for an artificial variable is

(A) $+M$

(B) $-M$

(C) zero

(D) Both (B) and (C)

44. If the dual has an unbounded solution, then the primal has

(A) no feasible solution

(B) unbounded solution

(C) feasible solution

(D) Both (A) and (C)

45. To ensure best marginal increase in the objective function value, a resource value may be increased whose shadow price is comparatively

(A) larger

(B) smaller

(C) Neither (A) nor (B)

(D) Both (A) and (B)

46. If we were to use opportunity cost value for an unused cell to test opportunity, it should be

(A) equal to zero

(B) most negative number

(C) most positive number

(D) any value

47. The vector $\alpha = (2, -5, 3) \in V_3(R)$ is a linear combination of vectors $\alpha_1 = (1, -3, 2)$, $\alpha_2 = (2, -4, -1)$ and $\alpha_3 = (1, -5, 7)$. This is

(A) true

(B) false

(C) Neither (A) nor (B)

(D) sometimes true and sometimes false

48. The set

$S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (0, 1, 0)\}$

is

(A) not a basis set

(B) a basis set

(C) Both (A) and (B)

(D) None of the above

49. If the mapping $f : V_2(R) \to V_2(R)$ is defined as $f(x, y) = (x^3, y^3)$, then f is

(A) linear

(B) non-linear

(C) non-homogeneous

(D) None of the above

50. Let U and V be the vector spac over the field F and let T is a linea transformation from U into V. If is finite dimensional, then

(A) rank (T) – nullity (T) = dim $(U$

(B) rank (T) + dim (U) = nullity $(T$

(C) rank (T) + nullity (T) = dim $(U$

(D) Both (A) and (B)

51. Let V be the vector space of 2×2 matrices over R and let $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Let $T : V \to V$ be a linear transformation defined by $T(A) = AB - BA$, then the dimension of kernel k of T is

(A) 1 (B) 2

(C) 3 (D) 4

52. The characteristic polynomial of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

is

(A) $-\lambda^3 - 6\lambda^2 + 9\lambda - 4$

(B) $\lambda^3 + 6\lambda^2 + 9\lambda - 4$

(C) $\lambda^3 + 6\lambda^2 + 9\lambda + 4$

(D) $-\lambda^3 + 6\lambda^2 - 9\lambda + 4$

53. If λ is the characteristic value of an invertible matrix A, then $\frac{|A|}{\lambda}$ is the characteristic value of

(A) A

(B) transpose of A

(C) adj A

(D) $2A$

54. The modulus of eigenvalues of an orthogonal matrix is

(A) 0

(B) -1

(C) 1

(D) 2

55. If $f(x, y) = x^3 + y^3 - 3x - 12y + 20$, then $f(x, y)$ has minima at

(A) $(-1, -2)$

(B) $(1, 2)$

(C) $(1, -2)$

(D) $(0, 0)$

$1+8-3-24+20$

20

$1-8-3+24+20$

56. $\lim\limits_{x \to 1} 2^{\frac{1}{x-1}}$ is

(A) 0

(B) 1

(C) 2

(D) Does not exist

$2^{1/x-1} = t$

$\frac{1}{x-1} = \log_2 t$

57. The function $f(x) = \sin x$ is

(A) not continuous on R

(B) not differentiable on R

(C) uniformly continuous on R

(D) not uniformly continuous on R

58. If $f(x) = x \sin\left(\frac{1}{x}\right)$, $x \neq 0$, $f(0) = 0$ then

(A) $f(x)$ is differentiable at $x = 0$

(B) $f(x)$ is not differentiable at $x = 0$

(C) $f(x)$ is not continuous at $x = 0$

(D) None of the above

59. The series $\Sigma(-1)^{n-1}\frac{1}{n^p}$ is

(A) divergent for every value of p

(B) convergent for every value of p

(C) convergent for $p > 0$

(D) divergent for $p > 0$

60. The series $\{\sqrt{(n^3+1)} - \sqrt{n^3}\}$ is

(A) convergent

(B) divergent

(C) conditionally convergent

(D) absolutely convergent

61. The radius of convergence of the power series $\sum\left\{\frac{2^{-n}}{(1+in^2)}z^n\right\}$ is

(A) 1

(B) 2

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

62. The radius of convergence of the power series $\sum\left\{\frac{z^n}{n^n}\right\}$ is

(A) 0

(B) 1

(C) 2

(D) ∞

63. If $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic, then its harmonic conjugate is

(A) $\tan^{-1}\left(\frac{y}{x}\right)+c$

(B) $\cos^{-1}\left(\frac{y}{x}\right)+c$

(C) x^2+y^2+c

(D) $\sin^{-1}\left(\frac{y}{x}\right)+c$

64. The value of $\oint_C \frac{e^z}{z-1}dz$, where C is the circle $|z|=2$ with positive orientation, is

(A) πei

(B) $\frac{\pi}{2}ei$

(C) $2\pi ei$

(D) $\frac{3}{2}\pi ei$

$2\pi i$

$2\pi i e$

65. The value of $\oint_C \frac{e^{i\pi z}}{2z^2-5z+2}dz$, where C is the circle $|z|=1$ with positive orientation, is

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) π

(D) $2\pi i$

66. In case of Newton-Raphson method, the roots are

(A) real

(B) complex

(C) Neither (A) nor (B)

(D) Both (A) and (B)

67. Newton's backward interpolation formula gives the most appropriate result

(A) in the beginning of the interval

(B) in the last of the interval

(C) near the middle of the interval

(D) before the beginning of the interval

68. In Simpson's $\frac{1}{3}$ rule, the number of subintervals is

(A) only 2

(B) more than 2

(C) 2 or multiple of 2

(D) Both (A) and (B)

69. The common area to the circles $r = a$ and $r = (\sqrt{2})a\cos\theta$ is

(A) $a^2(\pi - 1)$

(B) $2a^2(\pi - 1)$

(C) $a^2(\pi + 1)$

(D) $\frac{a^2}{2}(\pi - 1)$

70. The partial differential equation by the elimination of a and b from $z = ax + by + ab$ is

(A) $z = p^2 + q^2$

(B) $z = px + qy$

(C) $z = px + qy + pq$

(D) $z = pq$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

71. The complete integral of $p^2 + q^2 = x + y$ is

(A) $z = (x + a)^2 + (y - a)^2 + b$

(B) $z = (x - a)^2 + (y + a)^2 + b$

(C) $z = (x + a)^{3/2} + (y - a)^{3/2} + b$

(D) $z = \frac{2}{3}(x + a)^{3/2} + \frac{2}{3}(y - a)^{3/2} + b$

where a and b are constants.

72. The particular integral of

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2y)$$

is

(A) $2x^2 \log(x + 2y)$

(B) $x^2 \log(x + 2y)$

(C) $x \log(x + 2y)$

(D) $\frac{1}{x + 2y}$

73. One root of the equation $x^3 - 8x - 2 = 0$ lies between

(A) 0 and 1

(B) 1 and 2

(C) 2 and 3

(D) 3 and 4

74. The value of $\Delta^3 (1-x)(1-2x)(1-4x)$ is

(A) -48

(B) 48

(C) -8

(D) 24

75. The unit normal to the surface $x^2 + 4y^2 - 3z^2 - 12 = 0$ at the point (3, 2, 1) is

(A) $\dfrac{3\hat{i} + 8\hat{j} - 3\hat{k}}{\sqrt{41}}$

(B) $\dfrac{3\hat{i} + \hat{j} + \hat{k}}{\sqrt{11}}$

(C) $\dfrac{3\hat{i} + 8\hat{j} - 3\hat{k}}{\sqrt{82}}$

(D) $3\hat{i} + 8\hat{j} - 3\hat{k}$

76. Which relation is ***not*** true for a common catenary?

(A) $s = c \sin \psi$

(B) $y = c \sec \psi$

(C) $x = c \log (\sec \psi + \tan \psi)$

(D) $y = c \cosh \left(\dfrac{x}{c}\right)$

77. If the radial and transverse velocities of a particle are always proportional to each other, the path is

(A) an equiangular spiral

(B) elliptic

(C) a catenary

(D) a cycloid

78. Two spheres of radii r_1 and r_2 intersect each other orthogonally. The radius of the common circle is

(A) $\dfrac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

(B) $\dfrac{r_1 + r_2}{\sqrt{r_1^2 + r_2^2}}$

(C) $\sqrt{r_1^2 + r_2^2}$

(D) $r_1 + r_2$

79. The angle between the pair of planes $2x^2 - 6y^2 - 12z^2 + 18yz + 2xz + xy = 0$ is

(A) $\tan^{-1}\left(\frac{16}{21}\right)$

(B) $\sec^{-1}\left(\frac{16}{21}\right)$

(C) $\cos^{-1}\left(\frac{16}{21}\right)$

(D) $\sin^{-1}\left(\frac{16}{21}\right)$

80. The value of p, so that the plane $x + 2y + 3z = p$ touches the central conicoid $x^2 - 2y^2 + 3z^2 = 2$, is

(A) 2

(B) 5

(C) 4

(D) 1

81. The number of independent components of a skew-symmetric tensor A_{ij} in an n-dimensional space is

(A) $\frac{n(n-1)}{2}$

(B) $\frac{n(n+1)}{2}$

(C) $n(n-1)$

(D) $(n+1)^2$

82. If A^i_{jk} is a tensor of type (1, 2) then the contracted tensor A^i_{ji} will be a tensor of type

(A) (0, 1)

(B) (1, 0)

(C) (2, 1)

(D) (1, 2)

83. For a simple dynamical system of n degrees of freedom, the Lagrangian L is a function of

(A) $\frac{n}{2}$ quantities

(B) n quantities

(C) $2n$ quantities

(D) None of the above

84. Any simple dynamical system moves in accordance with Lagrange's equations of the form

(A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) + \frac{\partial T}{\partial q_r} = 0$

(B) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = 0$

(C) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) + \frac{\partial L}{\partial q_r} = 0$

(D) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r} = 0$

85. The relation among operators Δ, ∇ and E is

(A) $E + \Delta = 1 = \nabla E$

(B) $E^{-1} = 1 + \Delta = \nabla\Delta$

(C) $\Delta = E - 1 = \nabla E$

(D) $E + 1 = \Delta = \nabla E$

86. Let $\langle a_n \rangle$ be a Cauchy sequence in a metric space (X, ρ) and let $\langle b_n \rangle$ be any sequence in X such that

$$\rho(a_n, b_n) < \frac{1}{n} \ \forall \ n \in N$$

then

(A) $\langle b_n \rangle$ is not a Cauchy sequence

(B) $\langle b_n \rangle$ is a Cauchy sequence

(C) $\langle b_n \rangle$ is divergent sequence

(D) None of the above

87. A metric space is compact iff it is

(A) totally bounded only

(B) complete only

(C) totally bounded and complete

(D) None of the above

88. The vector $\vec{v} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is

(A) solenoidal

(B) irrotational

(C) Both (A) and (B)

(D) rotational

89. The value of $\iint_S \vec{F} \cdot \hat{n} \, dS$, where S is the surface of the cube bounded by planes $x = 0$, $x = 1$; $y = 0$, $y = 1$; $z = 0$, $z = 1$ and $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, is

(A) 3

(B) 2

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

90. If

$u = x^2 + y^2 + z^2$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $\operatorname{div}(u\vec{r})$ in terms of u is

(A) $2u$

(B) $3u$

(C) $4u$

(D) $5u$

91. The solution of

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 2e^{3x}$$

is

(A) $y = (c_1 + c_2x)e^{3x} + x^2e^{3x}$

(B) $y = (c_1 + c_2x)e^{-3x} + 2e^{3x}$

(C) $y = (c_1 + c_2x)e^{3x} - \frac{xe^{3x}}{2}$

(D) $y = (c_1 + c_2x)e^{-3x} + x^2e^{3x}$

92. The solution of

$$y \log y \frac{dx}{dy} + x - \log y = 0$$

is

(A) $xy = \log y + c$

(B) $x \log y = \frac{1}{2}(\log y)^2 + c$

(C) $x^2 \log y = \frac{1}{2}(\log y)^2 + c$

(D) $y = (\log x)^2 + c$

93. The solution of

$$(2x \log x - xy)dy + 2ydx = 0$$

is

(A) $y \log x + y^2 = c$

(B) $2y \log x - y^2 = c$

(C) $2y \log x + \frac{y^2}{2} = c$

(D) $2y \log x - \frac{1}{2}y^2 = c$

94. The solution of

$$(x+y)^2\left(x\frac{dy}{dx} + y\right) = xy\left(1 + \frac{dy}{dx}\right)$$

is

(A) $xy = \frac{1}{x+y} + c$

(B) $\log xy + \frac{1}{x+y} = c$

(C) $\log xy - \frac{1}{x+y} = c$

(D) $xy + \frac{1}{x+y} = c$

95. The solution of $\cos(x+y)dy = dx$ is

(A) $y = \tan\left(\frac{x+y}{2}\right) + c$

(B) $y + \tan\left(\frac{x+y}{2}\right) = c$

(C) $(x+y)^2 + \tan x = c$

(D) None of the above

96. The value of $\iint xy\,dx\,dy$ over the region in the positive quadrant for which $x + y \leq 1$ is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{18}$

(D) $\frac{1}{24}$

97. The value of $\iiint\limits_R (x+y+z)\,dx\,dy\,dz$, where $R: 0 \leq x \leq 1$, $1 \leq y \leq 2$, $2 \leq z \leq 3$, is

(A) $\frac{9}{2}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{7}{2}$

98. The integral $\int_1^{\infty} \frac{\log x}{x^2} dx$ is

(A) divergent

(B) oscillatory

(C) convergent

(D) None of the above

99. The value of $\int_0^{\infty} x^2 e^{-x^2} dx$ is

(A) $\frac{\sqrt{\pi}}{2}$

(B) $\frac{\sqrt{\pi}}{4}$

(C) $\sqrt{\pi}$

(D) $\frac{3}{4}\sqrt{\pi}$

100. The value of $\int_0^1 \left\{\log\left(\frac{1}{x}\right)\right\}^{n-1} dx$ is

(A) $\overline{|n}$

(B) $\overline{|n+1}$

(C) $\frac{\overline{|n}}{2}$

(D) $\frac{\overline{|n+1}}{2}$