UPSC
IES/ISS

Previous Year Paper
(Statistics) Paper-I
22 June, 2024

I.E.S/ I.S.S. Examination, 2024

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

T.B.C. : SGSE–E–STS **Test Booklet Series**

B

Serial No. 1002994

TEST BOOKLET

STATISTICS

Paper—I

Time Allowed : Two Hours **Maximum Marks : 200**

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet.* You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

1. If $X_{(1)}, X_{(2)}, X_{(3)}, X_{(4)}, X_{(5)}$ are order statistics from a population with pdf $f(x) = 2e^{-2x}$, $x > 0$, then what is the density function of $X_{(1)}$?

(a) $10e^{-10X_{(1)}}$

(b) $1 - e^{-2X_{(1)}}$

(c) $1 - e^{-10X_{(1)}}$

(d) $10e^{-2X_{(1)}}$

Consider the following for the next **two (2)** items that follow :

Let $X_1, X_2, X_3, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$ distribution and

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2, \quad \bar{X} = \frac{1}{n}\sum_{i=1}^{n}X_i$$

2. What is the moment-generating function of S^2?

(a) $\left(1 - \frac{2(n-1)t}{\sigma^2}\right)^{-\frac{n-1}{2}}$

(b) $\left(1 - \frac{(n-1)t}{\sigma^2}\right)^{-\frac{n-1}{2}}$

(c) $\left(1 - \frac{2\sigma^2 t}{n-1}\right)^{-\frac{n-1}{2}}$

(d) $\left(1 - \frac{\sigma^2 t}{n-1}\right)^{-\frac{n-1}{2}}$

3. If

$$r = \frac{1}{2}\frac{\Gamma\left(\frac{n-3}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}; \quad n > 3$$

then what is the value of $E\left(\frac{1}{S^2}\right)$?

(a) $\frac{(n-1)r}{\sigma^2}$

(b) $\frac{nr}{\sigma^2}$

(c) $\frac{2nr}{\sigma^2}$

(d) $\frac{nr}{2\sigma^2}$

Consider the following for the next **three (3)** items that follow :

Let $X_1, X_2, X_3, ..., X_n$ be iid $U(0, 1)$ variates and $T = -\ln(X_1 X_2 X_3 ... X_n)$.

4. What is $E(T)$ equal to?

(a) n

(b) $2n$

(c) $4n$

(d) $n/2$

5. What is Var (T) equal to?

(a) $4n$

(b) $3n$

(c) $2n$

(d) n

6. What is the moment-generating function of T?

(a) $(1-2t)^{-2n}$

(b) $(1-t)^{-2n}$

(c) $(1-t)^{-n}$

(d) $(1-2t)^{-n/2}$

Consider the following for the next **four (4)** items that follow :

Let $X_1, X_2, X_3, \ldots, X_7$ be iid random variables, each having the $N(0, 1)$ distribution.

7. What is the distribution of the following?

$$\frac{\sqrt{2}(X_5 - X_6 + X_7)}{\sqrt{(X_1 + X_2 + X_3)^2 + 3X_4^2}}$$

(a) $N(0, 1)$

(b) $t_{(2)}$

(c) $t_{(3)}$

(d) $t_{(7)}$

8. What is $E\left(\frac{1}{X_1^2 + X_2^2 + X_3^2 + X_4^2}\right)$ equal to?

(a) 0

(b) 1

(c) 1/2

(d) 1/9

9. What is $E\left(\frac{X_1 + X_2 + X_3}{X_4^2 + X_5^2 + X_6^2 + X_7^2}\right)$ equal to?

(a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

10. If

$$\lambda\left(\frac{X_3 + X_4 + X_5 + X_6 + X_7}{X_1 + X_2}\right)$$

~ Cauchy (0, 1) distribution

then what is the value of $5\lambda^2$?

(a) 1

(b) 2

(c) 4

(d) 5

11. Consider the following pairs :

I. $x_1^2 + 2x_1x_2 + x_2^2;\ y_1^2 + 2y_1y_2 + 2y_2^2$

II. $x_1^2 - 2x_1x_2 - x_2^2;\ 2y_1^2 + y_1y_2 - y_2^2$

III. $x_1^2 + 2x_1x_2 - 2x_2^2;\ 2y_1^2 + y_1y_2 + 2y_2^2$

IV. $x_1^2 + x_1x_2 - x_2^2;\ y_1^2 + y_1y_2 - y_2^2$

How many of the above are non-orthogonal pairs?

(a) Only one pair

(b) Only two pairs

(c) Only three pairs

(d) All four pairs

12. Suppose X and Y are zero-mean jointly normally distributed random variables such that $\sigma_X^2 = 4$, $\sigma_Y^2 = \frac{17}{9}$, $E(XY) = 2$. If $Z = 2X - 3Y$, then what is the pdf of Z?

(a) $\frac{1}{\sqrt{12\pi}} e^{-\frac{z^2}{12}}$

(b) $\frac{1}{\sqrt{18\pi}} e^{-\frac{z^2}{18}}$

(c) $\frac{1}{\sqrt{6\pi}} e^{-\frac{z^2}{9}}$

(d) $\frac{1}{\sqrt{8\pi}} e^{-\frac{z^2}{8}}$

13. If X is a normally distributed random variable with $\mu = 1$ and $\sigma^2 = 2$, then the asymptotic distribution of sample median for $n = 20$ is

(a) $N(1, 1/10)$

(b) $N(1, \pi/20)$

(c) $N(1, \pi/10)$

(d) $N(1, \pi/2)$

14. Suppose X and Y are independent random variables having chi-square distributions with m and n degrees of freedom. Which of the following statements is/are correct?

I. $\frac{nX}{mY}$ follows $F_{m,\,n}$.

II. $\frac{mY}{nX}$ follows $F_{n,\,m}$.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

15. It is known that $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$ are two regression equations. Consider the following statements :

I. The regression coefficient of X on Y is 0·8.

II. The correlation coefficient is 0·6.

III. The most probable value of Y, when $X = 30$, is 30·6.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

16. The moment-generating function of X, when X follows $\chi^2(10)$, is

(a) $(1-t)^{-5}$

(b) $(1-t)^{-10}$

(c) $(1-2t)^{-5}$

(d) $(1-2t)^{-10}$

17. The pdf $g_n(x)$ of $X_{(n)}$ in a random sample of size n from the exponential distribution with pdf $f(x) = e^{-x}$, $x \geq 0$, is

(a) $(1-e^{-x})^{n-1}$

(b) $n(1-e^{-x})^{n-1}e^{-x}$

(c) $(1-e^{-x})^{n}$

(d) $n(1-e^{-x})^{n-1}$

18. If $4\sigma_x^2 = 9\sigma_y^2 = 25$, where x and y are independently distributed, then what is the value of $\text{Var}\left(\frac{2x+3y}{\sqrt{2}}\right)$?

(a) 25

(b) 50

(c) 62·5

(d) 75

19. If $\text{Cov}(2x, y) = 4$, $\text{Var}(x/4) = 1$ and $\text{Var}(y/9) = 7$, then what is the value of $(18r(x, y))^2$ equal to?

(a) 7

(b) 1/7

(c) 4

(d) 1/4

20. If $X_{(1)}$, $X_{(2)}$, $X_{(3)}$, $X_{(4)}$, $X_{(5)}$ are order statistics from $U[0, 2]$, then what is the value of the pdf $f_{X_{(1)}}(1)$?

(a) 1/16

(b) 5/16

(c) 1/32

(d) 5/32

21. A point (x, y) is to be selected from the square S containing all points (x, y) such that $0 \le x \le 1$ and $0 \le y \le 1$. What is the probability that the selected point belongs to subset of points such that $0 \cdot 5 < (x + y) < 1 \cdot 5$?

(a) $0 \cdot 15$

(b) $0 \cdot 25$

(c) $0 \cdot 45$

(d) $0 \cdot 75$

22. Let $(X_1, X_2, X_3, \ldots, X_n)$ be a random sample from $U(0, 1)$ distribution. $\ln X$ follows

(a) gamma distribution

(b) negative exponential distribution

(c) half-normal distribution

(d) None of the above

23. If X, Y and Z are iid $N(0, 1)$ variates, then the characteristic function of $\frac{X + YZ}{\sqrt{1 + Z^2}}$ is

(a) $e^{-\frac{t^2}{2}}$

(b) e^{-2t^2}

(c) $\frac{1}{1 + t^2}$

(d) $\frac{1}{\sqrt{1 + t^2}}$

24. Two random variables X and Y are such that $P(X = \pm 1) = 0 \cdot 5$ and $Y \sim N(0, 1)$. What is the characteristic function of $2XY$?

(a) $e^{-\frac{t^2}{2}}$

(b) e^{-2t^2}

(c) $\frac{1}{4 + t^2}$

(d) $\frac{1}{\sqrt{4 + t^2}}$

25. If X_1 and X_2 are iid $N(0, 1)$ variates, then the characteristic function of $X_1 X_2$ is

(a) e^{-t^2}

(b) $\frac{1}{1 + t^2}$

(c) $\frac{1}{\sqrt{1 + t^2}}$

(d) $e^{-2|t|}$

26. Let $U(t)$ and $V(t)$ be the moment-generating functions of two random variables X and Y respectively, and $V(t) = U(\log U(t))$. If $E(X) = 2$, then what is the value of $E(Y)$?

(a) 1

(b) 2

(c) 4

(d) $\ln 2$

Consider the following for the next **two (2)** items that follow :

Gambles are independent and each one results in the player being equally likely to win or lose 1 unit. Let W denote the net winnings of a gambler whose strategy is to stop gambling immediately after his first win.

27. If the first win is on trial N, then the winnings is

(a) $2 / N$ (b) $2 - N$

(c) $2N$ (d) $(2 - N) / N$

28. What is $P(W < 0)$ equal to?

(a) 1/2

(b) 1/4

(c) 3/4

(d) 0

Consider the following for the next **two (2)** items that follow :

Suppose that a balanced, six-faced dice is rolled repeatedly until the same number appears on two successive rolls and let X denote the number of rolls that are required.

29. What is the value of $P(X = 10)$?

(a) $\frac{5^8}{6^9}$ (b) $\frac{5^9}{6^{10}}$

(c) $\frac{5^{10}}{6^{10}}$ (d) $\frac{5^{11}}{6^{12}}$

30. What is the value of $E(X)$?

(a) 17

(b) 7

(c) 18/35

(d) 17/25

31. Two events A and B are such that $P(A) = 1/3$, $P(B) = 1/5$ and $P(A \mid B) + P(B \mid A) = 2/3$. What is the value of $12P(\overline{A} \cup \overline{B})$?

(a) 23/2

(b) 11

(c) 7

(d) 5

32. Let the joint distribution of X and Y be given by

$$f(x, y) = \begin{cases} cxye^{-(x^2+y^2)}; & x > 0, y > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

With reference to the above, consider the following statements :

I. The value of c for $f(x, y)$ to be a proper pdf is 4.

II. The probability $P(X^2 > 2Y^2)$ is 1/3.

III. X and Y are independently distributed.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

33. Let X and Y have joint probability mass function

$$P(X = x, Y = y) = \frac{e^{-2}}{x!(y-x)!};$$

$x = 0, 1, 2, \ldots,$ y and $y = 1, 2, 3, \ldots$

With reference to the above, consider the following statements :

I. Y follows Poisson distribution with parameter 1/2.

II. $E(X) = 1$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

34. The probability mass function of a random variable X is given by

$P(X = n) = 2^{-n}$; $n = 1, 2, 3, \ldots$

Define a random variable

$$Y = \begin{cases} -1, & \text{if } X \text{ is odd} \\ +1, & \text{if } X \text{ is even} \end{cases}$$

With reference to the above, consider the following statements :

I. $P(Y = 1) = 2/3$

II. $E(Y) = -1/3$

III. $V(Y) = 8/9$

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

35. Let X and Y be jointly distributed with joint probability mass function

$$P(x, y) = \frac{1}{2^{y+2}(y+1)}\left(\frac{2y+1}{2y+2}\right)^x; \quad x, y = 0, 1, 2, \ldots$$

With reference to the above, consider the following statements :

I. The marginal distribution of Y is geometric distribution.

II. Var $(Y) = 1$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

36. Let X and Y be jointly distributed as BN(1, 0, 36, 16, 1/2). Consider the following statements :

I. $E(Y \mid X = 7) = 2$

II. Cov $(X + Y, Y - X) = -20$

III. $V(X \mid Y = 1) = 27$

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

37. Let X and Y be two independent random variables having moment-generating functions

$$M_X(t) = \left(\frac{3}{4} + \frac{1}{4}e^t\right)^3 \text{ and } M_Y(t) = e^{2(e^t - 1)}$$

What is $P[(X+Y) < 2]$ equal to?

(a) $\frac{27}{64}e^{-2}$ *(b)* $\frac{27}{16}e^{-2}$

(c) $\frac{81}{32}e^{-2}$ *(d)* $\frac{27}{32}e^{-2}$

38. Let $x_1, x_2, \ldots, x_{10}$ be a sample of size 10 from an exponential distribution with mean θ. The lower bound to the probability $P(0 < \bar{X} < 2\theta)$, where

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$$

is

(a) 0·01 *(b)* 0·09

(c) 0·1 *(d)* 0·9

39. Suppose $Y = X^2$ and $f(x) = 1$, $0 < x < 1$. What is the pdf of Y?

(a) $\sqrt{y}$, $0 < y < 1$ *(b)* $\frac{\sqrt{y}}{2}$, $0 < y < 1$

(c) $\frac{1}{2\sqrt{y}}$, $0 < y < 1$ *(d)* y, $0 < y < 1$

40. If the moment-generating function of a distribution is

$$M_X(t) = \frac{\lambda}{\lambda - t} \text{ for } t < \lambda$$

then what are mean and variance of the distribution respectively?

(a) $\frac{-1}{\lambda - t}, \frac{\lambda^2}{(\lambda - t)^2}$ *(b)* $\frac{1}{\lambda}, \frac{1}{\lambda^2}$

(c) λ, λ^2 *(d)* $1, \lambda^2$

41. Consider the following statements in respect of debugger :

I. Debugger is a program used to detect errors and bugs in a program.

II. In order to debug the program, a debugger helps us perform step-by-step execution of a program.

III. In order to debug the program, a debugger helps us perform stopping the execution of the program until the errors are corrected.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

42. Consider the following statements in respect of Operating System :

I. It is the principal component of system software.

II. It is responsible for overall management of the computer resources.

III. It provides an interface between the computer and user and helps in implementing the application programs.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

43. Consider the following tasks :

I. Search and replace

II. Encryption

III. Virus scanner

IV. Backup

V. Data recovery

How many of the above are tasks of Utility Programs?

(a) Only two

(b) Only three

(c) Only four

(d) All five

44. Which of the following security services prevents either sender or receiver from denying a transmitted message?

(a) Authentication

(b) Confidentiality

(c) Non-repudiation

(d) Integrity

45. At which layer of TCP/IP do ICMP and ARP function?

(a) Network layer

(b) Application layer

(c) Transport layer

(d) Physical layer

46. What is the representation of – 59 in the 2's complement system?

(a) 11000101

(b) 11000100

(c) 10000101

(d) 11000001

47. Consider the following statements :

I. Both compiler and interpreter can translate the program written in high-level language into machine-level instructions.

II. The interpreter translates the whole program into machine language program before executing any of the instructions.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

48. Consider the following statements in respect of Webcam :

I. It is an input device.

II. The quality of the video taken through the Webcam largely depends on frame rate and resolution.

III. For a video to be smooth and clear, the frame rate is maximum and resolution is minimum.

Which of the statements given above is/are correct?

(a) II only

(b) I and II only

(c) I and III only

(d) I, II and III

49. Which of the following are main operations of CPU?

I. Fetching instructions from the memory

II. Decoding the instructions

III. Executing the instructions

IV. Storing the results back in the memory

Select the correct answer using the code given below.

(a) I, III and IV only

(b) I, II and IV only

(c) II and III only

(d) I, II, III and IV

50. In an e-mail service, the protocols used are SMTP, POP3 and IMAP. What are the port numbers in sequence?

(a) 110, 143, 25

(b) 25, 143, 110

(c) 143, 110, 25

(d) 25, 110, 143

51. What are the functions of BIOS in computer system?

I. It performs power-on self-test.

II. It boots the computer system.

III. It provides hardware-independent access to the physical devices.

Select the correct answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

52. Sending messages using IPSEC protocol on the internet between two IPSEC compliant router servers is also termed as

(a) tunneling

(b) leased line

(c) packet filtering

(d) message switching

53. Which one of the following methods is used for interprocess communication in distributed systems?

(a) Shared cache memory

(b) Message passing

(c) Shared region

(d) Cache memory

54. Consider the following statements :

I. The hexadecimal equivalent of $(1101010101)_2$ is 355.

II. The complement of $(47)_{10}$ is $(53)_{10}$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

55. Which of the following are network layer protocols?

I. HTTP

II. FTP

III. NMP

IV. SMTP

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

56. Which of the following are network connecting devices?

I. Repeater

II. Bridge

III. Hub

IV. Bluetooth

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

57. Consider the following statements regarding Database Management System (DBMS) :

I. It helps the user in maintaining data integrity while performing the various operations on the database values.

II. It helps the user in maintaining consistency in the information stored in the database.

III. It allows the user to maintain the security of the confidential information by password-protecting the database.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

58. Consider the following statements :

I. Light pen is an electro-optical pointing device which is generally connected to the Visual Display Unit (VDU) of the computer.

II. Touchscreen is a pointing device that enables us to enter data such as text, pictures and images by directly touching the screen.

III. Joystick is a pointing device which controls the movement of the cursor on the screen by pointing in a particular direction only.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

59. Which of the following are looping statements in general of a computer programming language?

I. While II. Break

III. For IV. Continue

V. Do while

Select the correct answer using the code given below.

(a) I and II

(b) II and IV only

(c) II, III and IV

(d) I, III and V

60. In which of the following data models are the records arranged in the form of a tree?

(a) Entity model

(b) Relational model

(c) Hierarchical model

(d) Network model

61. If y satisfies the initial value problem

$$\frac{dy}{dx} = 1 + xy; \quad y(2) = 0$$

then what is the approximation of y at $x = 3$? (Use second approximation of Picard's method)

(a) 3·0

(b) 2·33

(c) 1·33

(d) 0

62. Consider the following statements :

I. Forward Euler method and backward Euler method for initial value problem have same order.

II. Classical fourth order Runge-Kutta method for initial value problem has higher order than forward Euler method.

III. Classical fourth order Runge-Kutta method for initial value problem has higher order than backward Euler method.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

63. Use Simpson's one-third rule to evaluate $\int_1^3 x^4 dx$ by dividing [1, 3] into two parts.

What is the value of the error?

(a) 0·12

(b) 0·19

(c) 0·27

(d) 0·33

64. Runge-Kutta method of order four is used to find the solution of the differential equation

$$\frac{dy}{dx} = f(x); \quad y(0) = 0$$

with step size h. If the solution is

$$y(h) = h\left[af(0) + bf\left(\frac{h}{2}\right) + cf(h)\right]$$

then what are the values of a, b, c respectively?

(a) 1/6, 2/3, 2/3

(b) 2/3, 2/3, 1/6

(c) 2/3, 1/6, 1/6

(d) 1/6, 2/3, 1/6

65. $y = x^2 + 2x + 3$ and y_0, y_1, y_2 are the values of y for $x = a$, $a + h$, $a + 2h$ respectively. What is the value of $\int_a^{a+2h} y\,dx$? (Use trapezoidal rule)

(a) $\frac{h}{2}(y_0 + 2y_1 + y_2)$

(b) $\frac{h}{2}(2y_0 + y_1 + 2y_2)$

(c) $\frac{h}{2}(y_0 + 2y_1 + 2y_2)$

(d) $\frac{h}{2}(y_0 + y_1 + y_2)$

66. What is the value of the following?

$$4y_0 + 6\Delta y_0 + 4\Delta^2 y_0 + \Delta^3 y_0$$

(a) $y_0 - y_1 + y_2 - y_3$

(b) $y_0 + y_1 + y_2 + y_3$

(c) $y_0 + y_1 - y_2 - y_3$

(d) $y_0 - y_1 - y_2 + y_3$

67. What is the value of $\nabla^5 y_n$?

(a) $\nabla^5 y_{n-5}$ *(b)* $\nabla^4 \Delta y_{n-5}$

(c) $\Delta^5 y_n$ *(d)* $\Delta^5 y_{n-5}$

68. To find the approximate value of $\int_a^b y\,dx$, where $b = a + 8h$, which of the following rules can be applied?

(a) Trapezoidal rule and Simpson's three-eighths rule

(b) Trapezoidal rule and Simpson's one-third rule

(c) Simpson's one-third rule and Weddle rule

(d) Simpson's three-eighths rule and Weddle rule

69. The value of the integral $\int_1^3 f(x)\,dx$ is 5 by using trapezoidal rule and 3 by using Simpson's one-third rule dividing [1, 3] into minimum number of intervals. What is the value of $f(2)$?

(a) 1/2

(b) 1

(c) 3/2

(d) 2

70. The third divided difference of a cubic polynomial is

(a) always constant but not necessarily zero

(b) always zero

(c) equal to sum of arguments

(d) equal to product of arguments

71. If $\Delta^2(ab^x) = \alpha b^x$, the interval of differencing being 1, then what is the value of α?

(a) $a(b-1)^2$

(b) a

(c) $(b-1)^2$

(d) $a(b-1)$

72. If $\left(\frac{\Delta^2}{E}\right)x^3 = \alpha x + \beta$, the interval of differencing being 1, then what are the values of α and β respectively?

(a) 6, 6

(b) 6, 0

(c) 0, 6

(d) 3, 3

73. If $\delta = \alpha \nabla E^n$, the interval of differencing being 1, then what are the values of α and n respectively?

(a) $1, -1/2$

(b) $1, 1/2$

(c) $1/2, -1/2$

(d) $-1/2, 1/2$

74. Which one of the following is the best possible choice for the step size h to tabulate $f(x) = \sin x$ on $[0, \pi/4]$ that results in truncation error for the linear interpolation at most 10^{-5}?

(a) $\pi/520$

(b) $\pi/320$

(c) $\pi/120$

(d) $\pi/92$

75. The roots of the least degree polynomial satisfied by (0, 1), (1/2, 3/2) and (1, 3) are

(a) real and distinct

(b) complex with real part zero

(c) real and repeated

(d) complex with non-zero real part

76. The bound on the truncation error of Lagrange linear interpolating polynomial $f(x)$ on $[a, b]$ is ε. What is the bound on the truncation error of Newton forward interpolating polynomial of $f(x)$ on $[a, b]$?

(a) $\varepsilon/2$

(b) ε

(c) 2ε

(d) 4ε

77. If the formula

$$\int_{x_0}^{x_2} f(x)\,dx = \alpha f(x_0) + \beta f(x_0 + h) + \gamma f(x_0 + 2h)$$

is exact for polynomial of as high order as possible for any $h > 0$, then what are the values of α, β, γ respectively?

(a) $\frac{h}{3}, \frac{4h}{3}, \frac{h}{3}$

(b) $\frac{4h}{3}, \frac{h}{3}, \frac{h}{3}$

(c) $h, \frac{h}{2}, h$

(d) $\frac{h}{2}, h, \frac{h}{2}$

78. For $\int_0^1 f(x)\,dx$, where $f(x)$ is cubic polynomial, consider the following statements :

I. Trapezoidal rule will give the exact value of the integral.

II. Simpson's one-third rule will give the exact value of the integral.

III. Simpson's three-eighths rule will give the exact value of the integral.

Which of the statements given above are correct?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

79. Consider the following data with regard to depth (y) at a distance (x) of a river 40 m wide :

Distance from one of the sides (x) :	0	10	20	30	40
Depth (y) at a distance (x) in metre :	0	4	7	9	12

What is the approximate area of cross-section of the river? (Use Simpson's one-third rule)

(a) 780 square metre

(b) 390 square metre

(c) 260 square metre

(d) 195 square metre

80. Consider the initial value problem $\frac{dy}{dx} = -8y$; $y(0) = 1$. What is the largest interval for step size h so that the forward Euler method to solve the initial value problem is absolutely stable?

(a) (0, 1/8)

(b) (0, 1/4)

(c) (0, 1/3)

(d) (0, 1/2)

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

★★★